

The aim of this mini-course is to explain recent developments in the theory of degenerate complex Monge-Ampère equations, with applications towards the construction of canonical metrics and the study of the Kähler-Ricci flow on mildly singular projective algebraic varieties.

Complex Monge-Ampère equations have been one of the most powerful tools in Kaehler geometry since Aubin and Yau's classical works, culminating in Yau's solution to the Calabi conjecture. A notable application is the construction of Kaehler-Einstein metrics on compact Kähler manifolds.

Whereas their existence on manifolds with trivial or ample canonical class was settled as a corollary of the Calabi conjecture, determining necessary and sufficient conditions on a Fano manifold to carry a Kähler-Einstein metric has only been solved recently by Chen-Donaldson-Sun.

Following Tsuji's pioneering work, degenerate complex Monge-Ampère equations have been intensively studied by many authors in the last decade. In relation to the Minimal Model Program, they led to the construction of singular Kähler-Einstein metrics with zero or negative Ricci curvature or, more generally, of canonical volume forms on compact Kaehler manifolds with nonnegative Kodaira dimension. Making sense of and constructing singular Kaehler-Einstein metrics on singular Fano varieties requires more advanced tools in the study of degenerate complex Monge-Ampère equations.

We shall survey the theory of Kähler-Einstein metrics/currents, explain their equivalent formulation in terms of degenerate complex Monge-Ampère equations, and present the variational approach developed by Berman-Boucksom-Guedj-Zeriahi, as well as its application to the study of the normalized Kaehler-Ricci flow. This allows in particular to generalize deep results of Perelman-Tian-Zhu. If time permits, we plan to discuss the most recent pluripotential parabolic approach of Guedj-Lu-Zeriahi.