The course will start with a recapitulation of the classical Brunn-Minkowski theorem and its functional version, Prékopa's theorem. Proofs will be based on the approach by Brascamp and Lieb, using a real variable version of Hormander's L^2-estimates.

Then the Prékopa-Leindler theorem (a partly non-convex version of Prékopa's theorem) will be discussed. The proof of this also has a complex counterpart, as it uses a real variable analog of Yau's solution to the Calabi conjecture.

After that, we review the Bergman kernel, and its interpretation as a metric on a line bundle, and give the first version of a complex Brunn-Minkowski theorem. Then we discuss direct images bundles, and different notions of positivity of their curvature. Finally, some applications to Kähler geometry will be sketched.