

This course will present the basic notions in the Hodge Theory of compact Hermitian and Kähler manifolds. It will start by introducing the notion of elliptic differential operators and by proving Garding's Inequality and its main geometric consequence on compact manifolds for these operators. The usual Laplace-type operators will then be introduced on both real and complex manifolds, they will be proved to be elliptic and the Hodge Isomorphism Theorem will then be deduced. No Kählerianity assumption will be needed so far.

The notion of Kähler metric on a complex manifold will then be introduced and examples of both Kähler (including projective) and non-Kähler manifolds will be provided. The Kähler commutation relations will then be proved and the resulting $\bar{\partial}$ -Lemma, Hodge Decomposition and Hodge Symmetry on compact Kähler manifolds will be deduced. If time permits, some basic notions in the Theory of Deformations of Complex Structures will also be presented along with the Kodaira-Spencer Semicontinuity Theorem in holomorphic families of compact complex manifolds.