# Data science in interdisciplinary research projects 

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## Who am I? Short resume

- A levels, Mathematics and Physics

1996

- Bachelor of Science, Industrial Mathematics

1998

- Master degree, Applied Mathematics

2002

- PhD in Applied Mathematics, Statistics

Comparison of estimation methods for nonlinear mixed effects models. Application to the modelling of the evolution of the leaf area index of crops observed by remote sensing

2003

2019

- Hired as a research engineer in Statistics at Université Toulouse III - Paul Sabatier

Habilitation Thesis
Fifteen years of applied research in data science

## What does a research engineer in statistics do?

- Officially (in French, data.enseignementsup-recherche.gouv.fr/pages/fiche_emploi__type_referens_iii_i_itrf/2refine.referens_ideE1044)
- engineer responsible for statistical aspects in a research laboratory
- manage statistical projects
- define a data collection and management plan and the associated processing chain
- participate in national and international research projects and associated publications
- ...
- In practice
- answer questions in the real world


## Address real-world problems

## HOW NOT TOBE WRONG

I blame word problems. They give a badly wrong impression of the relation between mathematics and reality. "Bobby has three hundred marbles and gives $30 \%$ of them to Jenny. He gives half as many to Jimmy as he gave to Jenny. How many does he have left?" That looks like it's about the real world, but it's just an arithmetic problem in a not very convincing disguise [...]

But real-world questions aren't like word problems. A real-world problem is something like "Has the recession and its aftermath been especially bad for women in the workforce, and if so, to what extent is this the result of Obama administration policies?" Your calculator doesn't have a button
THE HIDDEN for this. Because in order to give a sensible answer, you need to know more than just numbers.
[...]

It's only after you've started to formulate these questions that you take out the calculator. But at that point the real mental work is already finished. Dividing one number by another is mere computation; figuring out what you should divide by what is mathematics.

THE FUTURE OF DATA ANALYSIS ${ }^{1}$
By John W. Tukey
Princeton University and Bell Telephone Laboratories

Received July 1, 1961.
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The future of data analysis can involve great process, the overcoming of real difficulties, and the provision of a great service to all fields of science and technology. Will it? That remains to us, to our willingness to take up the the rocky road of real problems in preference to smooth road of unreal assumptions, arbitrary criteria, and abstract results without real attachments.

## The rocky road of real problems...



... smooth road of unreal assumptions, arbitrary criteria, and abstract results without real attachments

A real-world problem

## How is global warming affecting plant growth?

## Back to linear algebra

The biological sciences are today in the process of changing from being primarily descriptive to being very much quantitative. As a result, biologists find themselves confronted more and more with large amounts of numerical data [...]. But the mere collecting and recording of data achieve nothing; having been collected, they must be investigated to see what information may be contained concerning the biological problem at hand.[...]

Frequently, however, biologists have to subject their data to more complex calculations, requiring procedures that involve mathematical details beyond their general experience. In order to carry out the mathematics the biologist in this situation must either learn the procedures himself, or at least learn something of the language of mathematics, that he may communicate satisfactorily with the mathematician whose aid he enlists.
S.R Searle (1966)

Matrix Algebra for the biological sciences

## Eigen decomposition

## $\mathrm{M}=\mathrm{V} \wedge \mathrm{V}^{\prime}$

M a square matrix $\mathrm{n} \times \mathrm{n}$
V columns of V are the eigen vectors of M corresponding to the eigen values $\Lambda$ diagonal matrix of the eigen values $\boldsymbol{\lambda}_{i}$
$\boldsymbol{\lambda}$ is an eigen value of $\mathbf{M}$
$\Leftrightarrow$
$\exists$ one vector $\mathbf{v}$ (length $n$ ) such that $\mathbf{M v}=\boldsymbol{\lambda} \mathbf{v}$

## Singular Value Decomposition (SVD)

## $\mathrm{M}=\mathrm{U} \mathrm{V}^{\prime}$

M a rectangular matrix $m \times n$
columns of $U$ and V are, respectively, left and right singular vectors corresponding to singular values
$\Sigma \quad$ diagonal matrix of the singular values of $M$
$\sigma$ is a singular value of $\mathbf{M}$
$\exists 2$ vectors $\mathbf{u}$ (length $\mathbf{m}$ ) and $\mathbf{v}$ (length $\mathbf{n}$ ) such that $\mathbf{M v}=\boldsymbol{\sigma} \mathbf{u}$ and $\mathbf{M} \mathbf{u}=\boldsymbol{u} \mathbf{v}$

## Link between eigen and SVD decompositions

$\mathrm{M}=\mathrm{U} \mathrm{V}^{\prime}$ (SVD)
Let's compute : M'M and MM'

```
M'M = (U\SigmaV')'U\SigmaV' replace M with SVD decomposition
\[
=\mathrm{V}^{\prime} \mathrm{U}^{\prime} \mathrm{U} \Sigma \mathrm{~V}^{\prime} \quad(\mathrm{AB})^{\prime}=\mathrm{B}^{\prime} \mathrm{A}^{\prime}
\]
\[
=\mathrm{V} \Sigma^{\prime} \Sigma \mathrm{V}^{\prime} \quad \mathrm{U} \text { unit vector, } \mathrm{U}^{\prime} \mathrm{U}=\mathrm{UU} U^{\prime}=1
\]
```

$M M^{\prime}=U \Sigma V^{\prime}\left(U \Sigma V^{\prime}\right)^{\prime}=U \Sigma V^{\prime} V \Sigma^{\prime} U^{\prime}=U \Sigma \Sigma^{\prime} U^{\prime}$
We obtain the eigen decomposition of M'M and MM' with eigen values equal to the square of the singular values and eigen vectors respectively equal to left and right singular vectors.

## Linear algebra for statistics

Principal Component Analysis (PCA)

- X a nxp data matrix
- PCA is an orthogonal linear transformation that projects the data in a new coordinate system such that the greatest variance of the data lies on the first coordinate (first PC), the second greatest variance on the second PC and so on...
- It can be shown that :
$\rightarrow$ The greatest variance is the first eigen value of $X^{\prime} X$
$\rightarrow$ Transforming coordinates is done using the first eigen vector


## Linear algebra for statistics

The singular values (in $\Sigma$ ) are the square roots of the eigenvalues of the matrix X'X. Each eigenvalue is proportional to the portion of the "variance" (more correctly of the sum of the squared distances of the points from their multidimensional mean) that is associated with each eigenvector. The sum of all the eigenvalues is equal to the sum of the squared distances of the points from their multidimensional mean. PCA essentially rotates the set of points around their mean in order to align with the principal components. This moves as much of the variance as possible (using an orthogonal transformation) into the first few dimensions. The values in the remaining dimensions, therefore, tend to be small and may be dropped with minimal loss of information (see below). PCA is often used in this manner for dimensionality reduction. PCA has the distinction of being the optimal orthogonal transformation for keeping the subspace that has largest "variance" (as defined above).

PCA, principle

## Teasing: Would you use a cubic box to pack a fishing rod?

## PCA, principle



Do we need 3 dimensions to represent 'standard' individuals?
$=$

Do we need a cubic box to pack a fishing rod?

## PCA, toy example

| Id | s.g | c.g | w.g | w | h |
| :--- | ---: | ---: | ---: | :---: | :---: |
| I1 | 106.2 | 89.5 | 71.5 | 65.6 | 174.0 |
| I2 | 110.5 | 97.0 | 79.0 | 71.8 | 175.3 |
| I3 | 115.1 | 97.5 | 83.2 | 80.7 | 193.5 |
| I4 | 104.5 | 97.0 | 77.8 | 72.6 | 186.5 |
| I5 | 107.5 | 97.5 | 80.0 | 78.8 | 187.2 |
| I6 | 119.8 | 99.9 | 82.5 | 74.8 | 181.5 |
| I7 | 123.5 | 106.9 | 82.0 | 86.4 | 184.0 |
| I8 | 120.4 | 102.5 | 76.8 | 78.4 | 184.5 |
| I9 | 111.0 | 91.0 | 68.5 | 62.0 | 175.0 |
| I10 | 119.5 | 93.5 | 77.5 | 81.6 | 184.0 |
| I11 | 105.0 | 89.0 | 71.2 | 67.3 | 169.5 |
| I12 | 100.2 | 94.1 | 79.6 | 75.5 | 160.0 |
| I13 | 99.1 | 90.8 | 77.9 | 68.2 | 172.7 |
| I14 | 107.6 | 97.0 | 69.6 | 61.4 | 162.6 |
| I15 | 104.0 | 95.4 | 86.0 | 76.8 | 157.5 |
| I16 | 108.4 | 91.8 | 69.9 | 71.8 | 176.5 |
| I17 | 99.3 | 87.3 | 63.5 | 55.5 | 164.4 |
| I18 | 91.9 | 78.1 | 57.9 | 48.6 | 160.7 |
| I19 | 107.1 | 90.9 | 72.2 | 66.4 | 174.0 |
| I20 | 100.5 | 97.1 | 80.4 | 67.3 | 163.8 |

## PCA, toy example

## Raw data

| Id | $\mathbf{s . g}$ | c.g | w.g | w | h |
| :--- | ---: | ---: | ---: | :---: | :---: |
| I1 | 106.2 | 89.5 | 71.5 | 65.6 | 174.0 |
| I2 | 110.5 | 97.0 | 79.0 | 71.8 | 175.3 |
| I3 | 115.1 | 97.5 | 83.2 | 80.7 | 193.5 |
| I4 | 104.5 | 97.0 | 77.8 | 72.6 | 186.5 |
| I5 | 107.5 | 97.5 | 80.0 | 78.8 | 187.2 |
| I6 | 119.8 | 99.9 | 82.5 | 74.8 | 181.5 |
| I7 | 123.5 | 106.9 | 82.0 | 86.4 | 184.0 |
| I8 | 120.4 | 102.5 | 76.8 | 78.4 | 184.5 |
| I9 | 111.0 | 91.0 | 68.5 | 62.0 | 175.0 |
| I10 | 119.5 | 93.5 | 77.5 | 81.6 | 184.0 |
| I11 | 105.0 | 89.0 | 71.2 | 67.3 | 169.5 |
| I12 | 100.2 | 94.1 | 79.6 | 75.5 | 160.0 |
| I13 | 99.1 | 90.8 | 77.9 | 68.2 | 172.7 |
| I14 | 107.6 | 97.0 | 69.6 | 61.4 | 162.6 |
| I15 | 104.0 | 95.4 | 86.0 | 76.8 | 157.5 |
| I16 | 108.4 | 91.8 | 69.9 | 71.8 | 176.5 |
| I17 | 99.3 | 87.3 | 63.5 | 55.5 | 164.4 |
| I18 | 91.9 | 78.1 | 57.9 | 48.6 | 160.7 |
| I19 | 107.1 | 90.9 | 72.2 | 66.4 | 174.0 |
| I20 | 100.5 | 97.1 | 80.4 | 67.3 | 163.8 |

## Covariance matrix

$$
\begin{array}{rlllrc} 
& \mathbf{s . g} & \text { c.g } & \text { w.g } & \text { w } & \text { h } \\
\text { s.g } & 68.6 & 37.7 & 28.1 & 55.3 & 61.2 \\
\text { c.g } & 37.7 & 37.5 & 33.9 & 45.7 & 32.4 \\
\text { w.g } & 28.1 & 33.9 & 50.8 & 56.6 & 27.7 \\
\text { w } & 55.3 & 45.7 & 56.6 & 85.7 & 59.5 \\
\text { h } & 61.2 & 32.4 & 27.7 & 59.5 & 109.3 \\
68.6+37.5+50.8+85.7+109.3=351.9
\end{array}
$$

351.9 represents the quantity of information contained in the data.

## Eigen decomposition of the covariance matrix

| R> eigen(cov(dataBody)) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| eigen() decomposition |  |  |  |  |  |
| \$values |  |  |  |  |  |
| [1] | 255.7 | 60.2 | 23.5 | 8.6 | 4.0 |
| \$vectors |  |  |  |  |  |
|  | [,1] | [,2] | [,3] | [,4] | [,5] |
| [1, | -0.45 | 0.16 | 0.78 | 0.18 | -0.36 |
| [2, | -0.32 | -0.25 | 0.26 | -0.72 | 0.49 |
| [3 | -0.34 | -0.53 | -0.33 | -0.24 | -0.66 |
| [4 | ] -0.53 | -0.36 | -0.18 | 0.60 | 0.44 |
| [5 | ] -0.54 | 0.70 | -0.42 | -0.16 | -0.02 |

Coefficients of linear combinations or loadings

```
PC1 = 0.45*shoulder.g + 0.32*chest.g + 0.34*waist.g + 0.54*weight + 0.54*height
PC2 = -0.16*shoulder.g + 0.25*chest.g + 0.53*waist.g + 0.36*weight - 0.70*height
```


## Transorm the data

Centered data

$$
\text { Ex: -6.50 }=0.45^{\star}(-1.9)+0.32^{\star}(-4.7)+0.34^{\star}(-3.8)+0.54^{\star}(-5)+0.54^{\star}(-0.4)
$$

| Id | s.g | c.g | w.g | w | h |  |  | PC1 | PC2 | PC3 | PC4 | PC5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I1 | -1.9 | -4.7 | -3.8 | -5.0 | -0.4 |  | I1 | -6.50 | -4.48 | -0.37 | -1.03 | 1.27 |
| I2 | 2.4 | 2.8 | 3.7 | 1.2 | 0.9 |  | I2 | 4.40 | 2.04 | 0.81 | 1.87 | 1.38 |
| I3 | 7.0 | 3.3 | 7.9 | 10.1 | 19.1 |  | I3 | 22.66 | -5.94 | -6.18 | 0.11 | 1.97 |
| I4 | -3.6 | 2.8 | 2.5 | 2.0 | 12.1 | PC1 PC2 PC3 PC4 PC5 | I4 | 7.78 | -5.24 | -8.38 | 4.10 | -1.74 |
| I5 | -0.6 | 3.3 | 4.7 | 8.2 | 12.8 |  | I5 | 13.73 | -2.67 | -8.02 | 0.82 | -2.15 |
| I6 | 11.7 | 5.7 | 7.2 | 4.2 | 7.1 |  | I6 | 15.67 | -0.15 | 4.49 | 2.33 | 4.40 |
| I7 | 15.4 | 12.7 | 6.7 | 15.8 | 9.6 |  | I7 | 26.99 | 3.19 | 6.29 | 0.04 | -3.08 |
| I8 | 12.3 | 8.3 | 1.5 | 7.8 | 10.1 |  | I8 | 18.41 | -3.43 | 5.63 | 1.09 | -1.96 |
| 19 | 2.9 | -3.2 | -6.8 | -8.6 | 0.6 |  | 19 | -6.25 | -8.48 | 4.97 | 0.79 | 1.86 |
| I10 | 11.4 | -0.7 | 2.2 | 11.0 | 9.6 | Apply | I10 | 16.78 | -3.67 | 1.99 | -7.08 | 1.22 |
| I11 | -3.1 | -5.2 | -4.1 | -3.3 | -4.9 | loadings | I11 | -8.83 | -0.78 | 0.28 | -3.02 | 0.07 |
| I12 | -7.9 | -0.1 | 4.2 | 4.9 | -14.4 |  | I12 | -7.28 | 15.41 | -2.31 | -3.00 | -2.35 |
| I13 | -9.0 | -3.4 | 2.6 | -2.4 | -1.7 |  | 113 | -6.45 | 2.25 | -7.60 | 0.95 | 1.15 |
| I14 | -0.5 | 2.8 | -5.8 | -9.2 | -11.8 |  | I14 | -12.51 | 2.68 | 8.91 | 4.27 | -1.53 |
| I15 | -4.1 | 1.2 | 10.7 | 6.2 | -16.9 |  | I15 | -3.65 | 20.76 | -0.30 | -2.45 | 1.99 |
| I16 | 0.3 | -2.4 | -5.4 | 1.2 | 2.1 |  | I16 | -0.63 | -4.62 | 0.34 | -3.46 | -2.80 |
| I17 | -8.8 | -6.9 | -11.8 | -15.1 | -10.0 |  | I17 | -23.61 | -5.07 | 2.20 | 1.19 | -1.15 |
| I18 | -16.2 | -16.1 | -17.4 | -22.0 | -13.7 |  | I18 | -37.50 | -9.07 | -1.33 | -1.89 | -0.02 |
| I19 | -1.0 | -3.3 | -3.1 | -4.2 | -0.4 |  | I19 | -4.98 | -3.61 | 0.33 | -0.50 | 1.02 |
| I20 | -7.6 | 2.9 | 5.1 | -3.3 | -10.6 |  | I20 | -8.24 | 10.89 | -1.74 | 4.86 | 0.44 |

255.7 is the greatest variance we can obtain with a linear combination of the initial variables.

```
Mean 0 0 0 0 0
Var. 255.7 60.2 23.5 8.6 4.0 = 351.9
```


## Graphical outputs (1/4)



## Graphical outputs (2/4)

| Loadings |  | PC1 | PC2 |
| :--- | :--- | :--- | ---: |
|  | shoulder.g | 0.45 | -0.16 |
|  | chest.g | 0.32 | 0.25 |
|  | waist.g | 0.34 | 0.53 |
|  | weight | 0.54 | 0.36 |
|  | height | 0.54 | -0.70 |



## Graphical outputs (3/4)

| Id | S. 9 | C. 9 | W. 9 | W | h |  | PC1 | PC2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I1 | 106.2 | 89.5 | 71.5 | 65.6 | 174.0 | I1 | -6.50 | -4.48 |
| I2 | 110.5 | 97.0 | 79.0 | 71.8 | 175.3 | I2 | 4.40 | 2.04 |
| I3 | 115.1 | 97.5 | 83.2 | 80.7 | 193.5 | I3 | 22.66 | -5.94 |
| I4 | 104.5 | 97.0 | 77.8 | 72.6 | 186.5 | I4 | 7.78 | -5.24 |
| I5 | 107.5 | 97.5 | 80.0 | 78.8 | 187.2 | I5 | 13.73 | -2.67 |
| I6 | 119.8 | 99.9 | 82.5 | 74.8 | 181.5 | I6 | 15.67 | -0.15 |
| I7 | 123.5 | 106.9 | 82.0 | 86.4 | 184.0 | I7 | 26.99 | 3.19 |
| I8 | 120.4 | 102.5 | 76.8 | 78.4 | 184.5 | I8 | 18.41 | -3.43 |
| 19 | 111.0 | 91.0 | 68.5 | 62.0 | 175.0 | I9 | -6.25 | -8.48 |
| I10 | 119.5 | 93.5 | 77.5 | 81.6 | 184.0 | I10 | 16.78 | -3.67 |
| I11 | 105.0 | 89.0 | 71.2 | 67.3 | 169.5 | I11 | -8.83 | -0.78 |
| I12 | 100.2 | 94.1 | 79.6 | 75.5 | 160.0 | I12 | -7.28 | 15.41 |
| I13 | 99.1 | 90.8 | 77.9 | 68.2 | 172.7 | I13 | -6.45 | 2.25 |
| I14 | 107.6 | 97.0 | 69.6 | 61.4 | 162.6 | I14 | -12.51 | 2.68 |
| I15 | 104.0 | 95.4 | 86.0 | 76.8 | 157.5 | I15 | -3.65 | 20.76 |
| I16 | 108.4 | 91.8 | 69.9 | 71.8 | 176.5 | I16 | -0.63 | -4.62 |
| 117 | 99.3 | 87.3 | 63.5 | 55.5 | 164.4 | 117 | -23.61 | -5.07 |
| I18 | 91.9 | 78.1 | 57.9 | 48.6 | 160.7 | I18 | -37.50 | -9.07 |
| I19 | 107.1 | 90.9 | 72.2 | 66.4 | 174.0 | I19 | -4.98 | -3.61 |
| I20 | 100.5 | 97.1 | 80.4 | 67.3 | 163.8 | I20 | -8.24 | 10.89 |
| $\operatorname{cor}(\mathrm{s.g}, \mathrm{PC} 1)$ $=0.87$  PC1 PC2 <br> $\operatorname{cor}(\mathrm{s.g}, \mathrm{PC} 2)$ $=0.15$  shoulder.g 0.87 <br>   chest.g 0.15  <br>   0.84 0.32  <br> $\operatorname{cor}(\mathrm{c} . \mathrm{g}, \mathrm{PC} 1)$ $=0.84$ waist.g 0.75 0.58 <br> $\operatorname{cor}(\mathrm{c} . \mathrm{g}, \mathrm{PC} 2)$ $=0.32$ weight 0.92 0.30 <br> $\ldots$  height 0.83 -0.52 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

Correlation circle plot


## Graphical outputs（4／4）

## Biplot

```
Id s.g c.g w.g w h
l1 106.2 89.5 71.5 65.6 174.0
I2 110.5
I3 115.1
I4 104.5
I5 107.5
l6 119.8 
I7 123.5 106.9 82.0 86.4 184.0
I8 120.4 102.5 76.8 78.4 184.5
I9 111.0 91.0 68.5 62.0 175.⿳亠丷⿵冂⿱丷口心
l10 119.5 93.5 77.5 81.6 184.%
I11 105.0 89.0
I12 100.2 94.1 79.6 75.5 160.0
I13 99.1 90.8 77.9 68.2 172.7
I14 107.6 97.0 69.6 61.4 162.6 6
I15 104.0 95.4 86.0
l16 108.4 91.8 69.9 71.8 176.5
I17 99.3 87.3 63.5 55.5 164.4
I18 91.9 78.1 57.9 48.6 160.7
I19 107.1 90.9 72.2 66.4 174.0
I20 100.5 97.1 80.4 67.3 163.8
```



```
Mean 108.1 94.2 75.3 70.6 174.4
```


## Focus on the variable plot

## Correlation $↔$ cosine

Remember trigonometry and right triangles:



The correlation between two variables is represented as:

- An acute angle $(\cos (\alpha)>0)$ if it is positive
- An obtuse angle $(\cos (\theta)<0)$ if it is negative
- A right angle $(\cos (\beta) \approx 0)$ if it is near zero


## Focus on the individual plot

- To interpret the graphical results of PCA must be done keeping in mind that one is looking at a projection on a plane (or in a volume for 3D representation)
- Be careful when interpreting visual proximities
- Illustration in comics with the only true super-heros ...

Scenario \& illustration: Pascal Jousselin Colour: Laurence Croix


## Focus on the individual plot

| I13 | 99.1 | 90.8 | 77.9 | 68.2 | 172.7 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| I14 | 107.6 | 97.0 | 69.6 | 61.4 | 162.6 |





## Graphical outputs: summary

Screeplot

- How many components?
- 90\% with 2 Pcs, 97\% with 3PCs, 100\% with 5PCs

Individual plot

- 'Natural' clusters, outliers...
- Caution: visual proximities

Variable plot, loading plot

- Correlation between variables
- Interpret components: PC1 « beefyness », PC2 «fatness, rotundity »


## PCA, simulated examples

Data set : 50 observations, 3 variables (V1-V2-V3)

## Case 1)

\{V1\}-\{V2\}-\{V3\}


Case 2)

$$
\{\mathrm{V} 1-\mathrm{V} 2\}-\{\mathrm{V} 3\}
$$



Pearson Correlation matrices

| 1) | V1 | V2 | V3 |
| :--- | :---: | :---: | ---: |
| V1 | 1.00 | -0.05 | -0.12 |
| V2 | -0.05 | 1.00 | 0.06 |
| V3 | -0.12 | 0.06 | 1.00 |

2) V1 V2 V3
$\begin{array}{llll}\text { V1 } & 1.00 & 0.90 & 0.08\end{array}$
V2 $0.90 \quad 1.00 \quad-0.01$
V3 0.08-0.01 1.00

Case 3)
\{V1-V2-V3\}

3) V1 V2 V3

V1 1.000 .930 .87
$\begin{array}{llll}\text { V2 } & 0.93 & 1.00 & 0.79\end{array}$
V3 $0.87 \quad 0.79 \quad 1.00$

PCA, simulated examples

Case 1)
Case 2)
Case 3)


## PCA, simulated examples

## Loadings

Dim. 1 Dim. 2 Dim. 3
V1 -0.23 0.14 0.07
V2 0.15 0.23 -0.03
V3 0.10-0.02 0.22



39.7\%
34.4\%



## PCA, simulated examples

## Loadings

Dim. 1 Dim. 2 Dim. 3
$\begin{array}{llll}\text { V1 } & 0.77 & 0.03 & 0.22\end{array}$
V2 0.97-0.06-0.17 V3 $0.05 \quad 0.91 \quad-0.02$




$\square$
cmp 3
$3.2 \%$



## PCA, simulated examples

## Loadings

|  | Dim. | Dim. 2 | Dim.3 |
| ---: | ---: | ---: | ---: |
| V1 | 1.07 | -0.05 | 0.22 |
| V2 | 1.23 | -0.34 | -0.13 |
| V3 | 1.07 | 0.44 | -0.07 |






## Extension to integration problems

- Multivariate unsupervised

One numerical dataset
Principal Component Analysis

## Extension to integration problems



## Generalized PLS, PLS-DA



$$
\text { . . . \} cij can be set by the user through a design matrix }
$$

## Sparsity

- High throughput experiments: too many variables, noisy or irrelevant depending on the goal aimed
- Some of the variable loadings, among the smallests, are set to 0 thanks to a LASSO (L¹) penalty
- Associated variables are not taken into account when calculating the PCs



## How is global warming affecting plant growth?

## WallOmics project

1/ Collect plants on the ground


2/ Gather seeds and grow them in controlled conditions (temperature, light, humidity...) at 2 different temperatures $\left(15^{\circ} \mathrm{C}\right.$ and $\left.22^{\circ} \mathrm{C}\right)$


3/ Collect different parts of the plant (stem, leaf, rosette...)


Pectin_RGI Pectin_HG XG Pectin_linearity Contribution_RG RGI_branching

|  | Pectin_RGI | Pectin_HG | XG | Pectin_linearity | Contribution_RG RGI_branching |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Col.22.1 | 75.96 | 60.29 | 92.88 | 0.94 | 0.29 | 2.70 |
| Col.22.2 | 63.71 | 76.68 | 89.76 | 1.32 | 0.17 | 3.49 |
| Col.22.3 | 69.05 | 78.73 | 103.20 | 1.28 | 0.20 | 2.92 |
| Col.15.1 | 57.56 | 43.65 | 81.75 | 0.85 | 0.20 | 4.95 |
| Col.15.2 | 79.39 | 74.34 | 116.76 | 1.03 | 0.16 | 4.92 |
| Col.15.3 | 84.36 | 73.31 | 123.27 | 0.96 | 0.17 | 5.18 |
| Roch.22.1 | 89.13 | 109.42 | 117.23 | 1.37 | 0.20 | 2.69 |
| Roch.22.2 | 120.02 | 138.92 | 135.48 | 1.33 | 0.24 | 2.16 |
| Roch.22.3 | 97.46 | 114.35 | 130.65 | 1.33 | 0.22 | 2.48 |
| Roch.15.1 | 91.94 | 88.57 | 136.65 | 1.07 | 0.19 | 4.04 |
| Roch.15.2 | 100.44 | 96.91 | 193.22 | 1.04 | 0.14 | 5.95 |
| Roch.15.3 | 96.42 | 97.84 | 179.30 | 1.09 | 0.13 | 6.07 |
| Grip.22.1 | 97.44 | 119.20 | 113.23 | 1.38 | 0.21 | 2.50 |
| Grip.22.2 | 90.28 | 88.47 | 111.65 | 1.12 | 0.24 | 2.76 |
| Grip.22.3 | 45.95 | 54.63 | 58.89 | 1.29 | 0.14 | 4.47 |
| Grip.15.1 | 77.22 | 72.26 | 99.00 | 1.01 | 0.14 | 6.26 |
| Grip.15.2 | 80.55 | 77.47 | 122.85 | 1.04 | 0.14 | 6.08 |
| Grip.15.3 | 86.40 | 82.43 | 132.43 | 1.03 | 0.13 | 6.24 |

4/ Analyze biological samples using high-throughput bio-technologies (DNA sequencing, mass spectrometry...)


5/ Generate very large datasets (thousands of features for each biological sample)
$\rightarrow$ need for statistical skills to analyze them

## WallOmics project: datasets

- R package WallomicsData

CRAN.R-project.org/package=WallomicsData

Generally, data integration can be defined as the process of combining data residing in diverse sources to provide users with a comprehensive view of such data. There is no universal approach to data integration, and many techniques are still evolving.
Schneider, M. V., \& Jimenez, R. C. (2012). Teaching the Fundamentals of
Biological Data Integration Using Classroom Games. PLoS Computational Biology 8(12)

- 60 samples A. thaliana:
- 5 ecotypes (Col, Grip, Hern, Roc, Hosp)
- 2 temperatures (low, high)
- 2 organs (stem, rosette)
- 3 replicates

- 4 tables: proteomics, transcriptomics, metabolomics (sugar), phenomics



## WallOmics project: one specific question

## Can We Determine a Multi-omics Signature to Classify Ecotypes on the Basis of Floral Stem Data?

- Multi-omics: consider all the datasets (proteomics, transcriptomics, metabolomics, phenomics $\rightarrow$ multi-block analysis
- Signature: select the most relevant variables inside each dataset $\rightarrow$ sparsity
- Classify ecotypes: supervised method $\rightarrow$ Discriminant Analysis
- Floral stem: filter data 'organ = stem’


## WallOmics project: method

## Multi-Block <br> Sparse <br> Projection to Latent Structures Discriminant Analysis

Systems biology
DIABLO: an integrative approach for identifying key molecular drivers from multi-omics assays
Amrit Singh ${ }^{1}$, Casey P. Shannon ${ }^{1}$, Benoît Gautier ${ }^{2}$, Florian Rohart ${ }^{3}$, Michaël Vacher ${ }^{4}$, Scott J. Tebbutt ${ }^{1}$ and Kim-Anh Lê Cao ${ }^{5, *}$

s.t. $\left\|a_{b}^{(q)}\right\|_{2}=1$ and $\left\|a_{b}^{(q)}\right\|_{1} \leq \lambda^{(q)}$ for all $1 \leq q \leq Q$

## WallOmics project: results



Clustered Image Map / heatmap


Correlation circle plot


Circos plot

## WallOmics project: publication

## Bintandin

Biontormatics
me

Volume 22, Issue 3

Journal article
A powerful framework for an integrative study with heterogeneous omics data: from univariate statistics to multi-block analysis Harold Duruflé, Merwann Selmani, Philippe Ranocha, Elisabeth Jamet, Christophe Dunand © Sébastien Déjean

Plant Science

## Cell wall modifications of two Arabidopsis

thaliana ecotypes, Col and Sha, in response to sub-optimal growth conditions: An integrative study

 8 Christophe Dunand ${ }^{2} \Omega$ s

- Harold: PhD student, vegetal biology
- Merwann: intern, applied mathematics
- Philippe: researcher biology
- Elisabeth: professor in biology
- Christophe: professor in biology, Harold's supervisor
- Sébastien : statistician, Harold's cosupervisor

Phenotypic Trait Variation as a Response to Altitude-Related Constraints in Arabidopsis Populations frontiers

Frontiers in Plant ScienceHarold Durufle ${ }^{\text {1t }}$. Philippe Ranocha ${ }^{1+}$, Duchesse Lacour Mbadinga Mbadinga ${ }^{1}$, Sébastien Déjean ${ }^{2}$, Maxime Bonhomme ${ }^{1}$, (a) Hélène San Clemente
Julio Sáez-Vásquez ${ }^{3.4}$, (3) Jean-Philippe Reichheld ${ }^{3}$, (a) Nathalie Escaravage ${ }^{5}$,

Article Menu

An Integrative Study Showing the Adaptation to Sub-Optimal Growth An integrative Study Showing the Adaptation to Sub-Optimal Growth Cell Wall Changes

 QChristophe Dunand $1^{1,}$,

## Other problems in the real world where maths can help

## Maths help... to give you a shining smile

Does wearing dental braces for 18 months really work?

For a shining smile...

... you must go through this...

... and maybe also that!
Modeling the dental arch with a 4 degre polynom without odd degres terms (for axial symetry)

$$
Y=b_{0}+b_{2} X^{2}+b_{4} X^{4}
$$

$M$. Rotenberg, Modélisation de la forme d'arcade dentaire
de jeunes adultes Www. theses.fr/1996T0U30012

## Maths help... to understand cheese ripening

What are the microbiological mechanisms involved in the cheese ripening process?


Complex microbial Ecosystems MUItiScale modElling: mechanistic and data driven approaches integration
www.itn-emuse.com
10 Europeans partners...

... to better understand the cheese ripening process

[^0]
## Maths help... to understand cheese ripening

PhD thesis (ongoing): Kernel approaches for the integration of biological data from heterogeneous sources


## Maths help... to optimize performance in sport



- Recruit new players
- Prevent injury
- Model collective behaviour
- Identify optimal strategies


Other domains


Differences in bread protein digestibility traced to wheat cultivar traits


## 888 cancers

Quantitative Analysis of Cell Aggregation Dynamics Identifies HDAC Inhibitors as Potential Regulators of Cancer Cell Clustering



Knowl Inf Syst (2012) 30:693-713
DOI 10.1007/10115-011-0391-7
REGULAR PAPER

How many performance measures to evaluat information retrieval systems?
$\underset{\text { Josiane Mothe }}{\text { Alain Baccini }}$


Received: 15 September $2010 /$ Revised: 4 January 2011 / Accepted: 30 January 2011 ,
Published © Soringer-Verlas London Limited 2011


The American Journal of CLINICAL NUTRITION

Gene expression profiling of human skeletal muscle in response to stabilized weight loss ${ }^{1}$

Dominique Larrouy, Pierre Barbe, Carine Valle, sebastien Dejean, Veronique Pelloux, Claire Thalamas, Jean-Philippe Bastard, Anne Le Bouil, Bertrand Diquet, Karine Clément, Dominique Langin, Nathalie Viguerie $\circ$ ㅇ
Frontiers


Urinary Amine and Organic Acid
Metabolites Evaluated as Markers for Childhood Aggression: The ACTION Biomarker Study
Support media can steer methanogenesis in the presence of phenol through biotic and abiotic effects

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& + \text { Add to Mendeley of Share } \% \text { Cit }
\end{aligned}
$$


open access




## When Bigger Is Better: 3D RNA Profiling of the Developing

 Head in the Catshark Scyliorhinus canicula
Hélène Mayeur ${ }^{1}$, Maxence Lanoizelet ${ }^{1}$, $+{ }^{4}{ }^{4}$. Sébastien Dejean ${ }^{5}$, Patrick Blader ${ }^{2}$,Sylvie Mazan ${ }^{1++}$ and Ronan Lagadec ${ }^{1 \dagger}$


[^0]:    "This presentation is part of a project that has received funding from the European Union's Horizon 2020
    research and innovation program under the Marie Skłodowska-Curie grant agreement No 956126

