```
# -*- coding: utf-8 -*-
"""
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"""

from matplotlib.pyplot import *
from math import *
from numpy import *
from numpy.random import *

import matplotlib.pyplot as plt









# Estimation de los  indices
  N=1000000
  a=array([2., 1.,4. ,3.])
  X=normal(0,1,(N,6))
  Y=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,4]+a[3]*X[:,0]*X[:,2]
  Y1=a[0]*X[:,0]+a[1]*X[:,3]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,3]
  Y2=a[0]*X[:,1]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,1]*X[:,2]
  Y3=a[0]*X[:,1]+a[1]*X[:,3]+a[2]*X[:,4]+a[3]*X[:,1]*X[:,3]
  Y12=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,2]
  S1N=(mean(Y*Y1)-mean(Y)*mean(Y1))/(mean(Y*Y)-mean(Y)**2)
  S2N=(mean(Y*Y2)-mean(Y)*mean(Y2))/(mean(Y*Y)-mean(Y)**2)
  S3N=(mean(Y*Y3)-mean(Y)*mean(Y3))/(mean(Y*Y)-mean(Y)**2)
  S12N=(mean(Y*Y12)-mean(Y)*mean(Y12))/(mean(Y*Y)-mean(Y)**2)
  S1=a[0]**2/sum(a*a)
  S2=a[1]**2/sum(a*a)
  S3=a[2]**2/sum(a*a)
  S12=(a[0]**2+a[1]**2+a[3]**2)/sum(a*a)




  #convergencia casi sugura
  N=500000
  h=400000
  NN=np.arange(1,N+1)
  Nplot=np.arange(1,h)
  a=array([2., 1.,4. ,3.])
  S1=a[0]**2/sum(a*a)
  S2=a[1]**2/sum(a*a)
  S3=a[2]**2/sum(a*a)
  S12=(a[0]**2+a[1]**2+a[3]**2)/sum(a*a)

  X=normal(0,1,(N,6))
  Y=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,4]+a[3]*X[:,0]*X[:,2]
  Y1=a[0]*X[:,0]+a[1]*X[:,3]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,3]
  Y2=a[0]*X[:,1]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,1]*X[:,2]
  Y3=a[0]*X[:,1]+a[1]*X[:,3]+a[2]*X[:,4]+a[3]*X[:,1]*X[:,3]
  Y12=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,2]

  NUM1=(cumsum(Y*Y1)/NN-cumsum(Y)/NN*cumsum(Y1)/NN)
  DEN1=(cumsum(Y*Y)/NN-(cumsum(Y)/NN)**2)
  S1N=NUM1/DEN1
  S1N1=S1N[N-h:N-1]
  Lim1=S1*ones(h-1)

  NUM2=(cumsum(Y*Y2)/NN-cumsum(Y)/NN*cumsum(Y2)/NN)
  DEN2=(cumsum(Y*Y)/NN-(cumsum(Y)/NN)**2)
  S2N=NUM2/DEN2
  S2N1=S2N[N-h:N-1]
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 Lim2=S2*ones(h-1)

 NUM3=(cumsum(Y*Y3)/NN-cumsum(Y)/NN*cumsum(Y3)/NN)
 DEN3=(cumsum(Y*Y)/NN-(cumsum(Y)/NN)**2)
 S3N=NUM3/DEN3
 S3N1=S3N[N-h:N-1]
 Lim3=S3*ones(h-1)


 NUM12=(cumsum(Y*Y12)/NN-cumsum(Y)/NN*cumsum(Y12)/NN)
 DEN12=(cumsum(Y*Y)/NN-(cumsum(Y)/NN)**2)
 S12N=NUM12/DEN12
 S12N1=S12N[N-h:N-1]
 Lim12=S12*ones(h-1)

 subplot(2,2,1)
 plot(S1N1)
 plot(Lim1)
 title('Convergence of S1N')
 xlabel('N')
 ylabel('S1N')

 subplot(2,2,2)
 plot(S2N1)
 plot(Lim2)
 title('Convergence of S2N')
 xlabel('N')
 ylabel('S2N')

 subplot(2,2,3)
 plot(S3N1)
 plot(Lim3)
 title('Convergence of S3N')
 xlabel('N')
 ylabel('S3N')

 subplot(2,2,4)
 plot(S12N1)
 plot(Lim12)
 title('Convergence of 12N')
 xlabel('N')
 ylabel('S12N')

  #Illustration del TCL para S1
 n=1000
 a=array([2., 1.,4. ,3.])
 N=10000
 SS1N=zeros(n)
 SS2N=zeros(n)
 SS3N=zeros(n)
 SS12N=zeros(n)
 for i in range(n):

      X=normal(0,1,(N,6))
      Y=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,4]+a[3]*X[:,0]*X[:,2]
      Y1=a[0]*X[:,0]+a[1]*X[:,3]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,3]
      Y2=a[0]*X[:,1]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,1]*X[:,2]
      Y3=a[0]*X[:,1]+a[1]*X[:,3]+a[2]*X[:,4]+a[3]*X[:,1]*X[:,3]
      Y12=a[0]*X[:,0]+a[1]*X[:,2]+a[2]*X[:,5]+a[3]*X[:,0]*X[:,2]
      SS1N[i]=(mean(Y*Y1)-mean(Y)*mean(Y1))/(mean(Y*Y)-mean(Y)**2)
     # SS2N[i]=(mean(Y*Y2)-mean(Y)*mean(Y2))/(mean(Y*Y)-mean(Y)**2)
     # SS3N[i]=(mean(Y*Y3)-mean(Y)*mean(Y3))/(mean(Y*Y)-mean(Y)**2)
     # SS12N[i]=(mean(Y*Y12)-mean(Y)*mean(Y12))/(mean(Y*Y)-mean(Y)**2)



 S1=a[0]**2/sum(a*a)
# S2=a[1]**2/sum(a*a)
# S3=a[2]**2/sum(a*a)
# S12=(a[0]**2+a[1]**2+a[3]**2)/sum(a*a)
```

```
 S1N=sqrt(N)*(SS1N-S1)
# S2N=sqrt(N)*(SS2N-S2)
# S3N=sqrt(N)*(SS3N-S3)
# S12N=sqrt(N)*(SS12N-S12)


 hist(S1N, bins  = linspace(-4,4,100), normed = True);
```