1

exo 31

$$z_1 = z_2 \iff \begin{cases} a^2 + a = 3a^2 - 3 \\ b^2 + 1 = 2b \end{cases}$$ partie réelle

$$\iff \begin{cases} 2a^2 - a - 3 = 0 & (L_1) \\ b^2 - 2b + 1 = 0 & (L_2) \end{cases}$$

Par (L_1), $\Delta = 25 > 0$ il y a deux solutions

$a_1 = -1$ et $a_2 = \frac{3}{2}$

Par (L_2), on trouve $b = 1$ (Id remarquable)

Ainsi $z_1 = z_2 \iff a \in \{-1 ; \frac{3}{2}\}$ et $b = 1$

exo 32

1) $z_1 + z_2 = x^2 + 2x - 3 + i(2x^2)$

$z_1 + z_2 \in i\mathbb{R} \iff \mathrm{Re}(z_1 + z_2) = 0 \iff x^2 + 2x - 3 = 0$

$\iff x_1 = -3$ ~~et~~ ou $x_2 = 1$ (car $\Delta = 16 > 0$)

2). $z_1 + z_2 \in \mathbb{R} \iff \mathrm{Im}(z_1 + z_2) = 0 \iff x = 0$

exo 37

1) . $\bar{z} = -i(2-2i) + 3i(1-2i) = -2 + 6 + i(-2+3)$

$= 4 + i$

2) $\bar{z} = 2i(1-i) + \frac{3}{2}i(2+4i)$

$= 2i + 2 - 3i + 6 = 8 - i$

3). $\bar{z} = (2-i)(1-3i) = -1 - 7i$

4) $\bar{z} = (-2i-3)(3-i) = -11 + 9i$

exo 38

1) $\bar{z} = (1-i)^2 = -2i$

2). $\bar{z} = (2-i)^3$

$= \cancel{8 + 4i}$

$= 8 - 12i - 6 + i$

$= 2 - 11i$

3). $\bar{z} = (1+i)^2(1+i)^2$

$= 2i \times 2i = -4$

4) $\bar{z} = (3-2i)^3$

$= 27 - 27i - 6 + 8i$

$= 21 - 19i$

exo 39

1) $\bar{z} = \frac{1}{-i} = i$

2) $\bar{z} = \frac{1}{1-i} = \frac{1+i}{2}$

3) $\bar{z} = \frac{2-i}{1+2i} = \frac{(2-i)(1-2i)}{5} = -i$

4) $\bar{z} = \frac{1+i}{-2i-1} = \frac{(1+i)(2i-1)}{5} = \frac{-3+i}{5}$

② 5). $\overline{z} = \dfrac{3-2i}{-2i-3} = \dfrac{(3-2i)(2i-3)}{13}$

$= \dfrac{-5+12i}{13}$

6). $\overline{z} = \dfrac{-1+i}{2-i} = \dfrac{(-1+i)(2+i)}{5} = \dfrac{-3+i}{5}$

108p40 Il suffit d'utiliser la règle du produit nul.

3) $(iz+1+i)(3iz-1)=0$

$\Leftrightarrow \left\{ \begin{array}{l} iz+1+i=0 \\ \quad \text{ou} \\ 3iz-1=0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} z = \dfrac{-1-i}{i} = \boxed{i-1} \\ \quad \text{ou} \\ z = \dfrac{1}{3i} = \boxed{-\dfrac{i}{3}} \end{array} \right.$

4) $[(1+i)z-1][(2+i)z+1]=0$

$\Leftrightarrow \left\{ \begin{array}{l} (1+i)z-1=0 \\ \quad \text{ou} \\ (2+i)z+1=0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} z = \dfrac{1}{1+i} = \boxed{\dfrac{1-i}{2}} \\ \quad \text{ou} \\ z = -\dfrac{1}{2+i} = \boxed{-\dfrac{2-i}{3}} \end{array} \right.$

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5) $\Delta = (-\sqrt{2})^2 - 4\times 1 = -2 < 0$

$z_1 = \dfrac{\sqrt{2}+i\sqrt{2}}{2}$ et $z_2 = \overline{z_1}$

6) $\Delta = (-6)^2 - 4\times 9\times 19$

$= -648 < 0$

$z_1 = \dfrac{6+18i\sqrt{2}}{18} = \dfrac{1}{3} + i\sqrt{2}$

et $z_2 = \overline{z_1}$