Introduction to particle methods and importance sampling/splitting methods
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The analysis of rare events is of great importance in many fields because of the risk associated to the event. Their probabilities are usually less than 10^{-9}. One can use many ways to study them : the fisrt one is statistical analysis, based on the standard extrem value distributions but needs a long observation period (see Aldous [**?**]), the second one is modelisation which leads to estimate the rare event probability either by analytical approach (see Sadowsky [**?**]), or by simulation.
In this course, we focus on the simulation approach based on Monte-Carlo method. Nevertheless crude simulation is impracticable for estimating such small probabilities : to estimate probabilities of order 10^{-10} with acceptable confidence would require the simulation of at least one thousand billion events (which corresponds to the occurrence of only one hundred rare events). To overcome these limits, fast simulation techniques are applied. In particular, importance sampling (IS) is a refinement of Monte-Carlo methods. The main idea of IS ([**?**]) is to make the occurrence of the rare event more frequent. More precisely IS consists in selecting a change of measure that minimizes the variance of the estimator. Another method is called splitting. The basic idea of splitting ([**?**]) is to partition the space-state of the system into a series of nested subsets and to consider the rare event as the intersection of a nested sequence of events. When a given subset is entered by a sample trajectory, random retrials are generated from the initial state corresponding to the state of the system at the entry point. More refined versions of splitting as particles systems ([**?**]) or RESTART ([**?**]) have been introduced in the last decades.