L1 CUPGE

Gpe 4 – Semaine 12 – TD 1

26 Avril 2021

A propos du test 8 :

$DL_{5,0}$ de $f(x) = \dfrac{\ln(1+x^3)}{1+x}$

$$\xrightarrow{} \ln(1+u) = u - \frac{u^2}{2} + o(u^2)$$

$$= \underbrace{\ln(1+x^3)}_{DL_{5,0}} \cdot \underbrace{\frac{1}{1+x}}_{DL_{5,0}}$$

$$DL_{5,0} \times DL_{5,0}$$

$$x^3 \underset{-}{\cancel{+}} \frac{x^6}{2} + o(x^6)$$

$$x^3 + o(x^5) \times \left(1 - x + x^2 \underset{\uparrow}{+} o(x^2)\right)$$

$$\cos(x) \underbrace{\ln(e^{x^2})}_{'' \; x^2} = x^2 \cos(x)$$

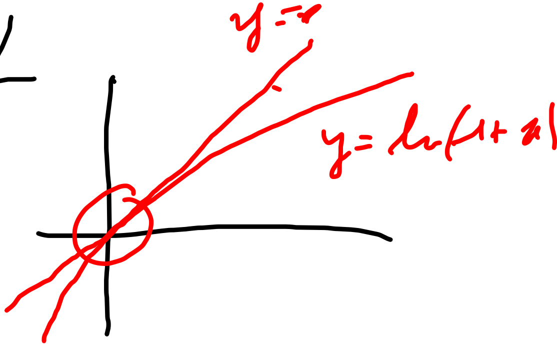
$$= x^2\left(1 - \frac{x^2}{2} + o(x^3)\right).$$

$$\frac{\ln(1+x)}{\sin(x)} = \frac{x+o(x)}{x+o(x)} = \frac{1+o(x)/x}{1+o(x)/x} \longrightarrow 1$$

$$\ln(1+x) \cancel{= x}$$ MAL

$$= x + \underbrace{o(x)}$$

$$\underset{0}{\sim} x$$



$f(x) \sim \underline{0}$ pas de sens

$$f \underset{a}{\sim} g$$

$$\frac{f(x)}{g(x)} \xrightarrow[x \to a]{} 1$$

$a_n \cancel{\sim} 0$ MAL

$f : P \in \mathbb{R}_2[X] \longrightarrow X^2 P'' \in \underset{\underline{\underline{2}}}{\mathbb{R}}[X]$

endo.

$d^\circ(X^2 P'') \leq 2$

$P \overset{\varphi}{\longmapsto} X^3 P''$

linéaire de $\mathbb{R}_2[X] \longrightarrow \mathbb{R}[X]$

Ce n'est pas un endo

$\varphi(X^2) \underline{\underline{= X^3 \cdot 2 \notin \mathbb{R}_2[X]}}$

Jeudi Matin : TD 10h à 12h

Possible à 11h
avant nae

Demain à ~~7h45~~ ~~8h30~~ 8h45

$\det(A) = 0 \iff A$ non <u>inversible</u>

Exercice : Calcul de $\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

(Feuille 12 exo 1)

à éviter

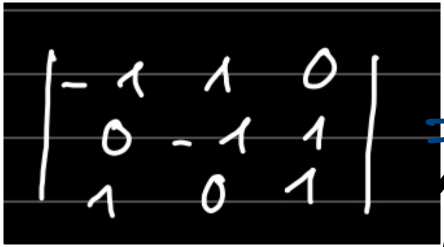
$$\underbrace{\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}}_{L_1} = -1\cdot(-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} + \underline{\underline{0}}\cdot(-1)^{2+1}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

dvp % ligne

$$+1(-1)^{3+1}\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}$$

$$= -1(-1.1 - 0.1) + 0 + 1(1-(-1)0)$$

$$= 1 + 1 = 2$$



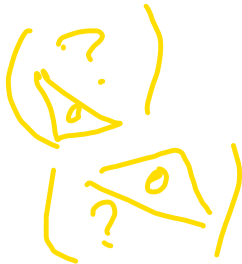
$$\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \underset{L_3 \leftarrow L_3 + L_1}{=} \begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \underset{\text{dvp \% L1}}{=} (-1)\cdot(-1)^{1+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(\overset{-2}{-2}) = 2$$

mieux

$$\underset{L_3 \leftarrow L_3 + L_2}{=} \begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = (-1)(-1)(2) = 2$$

Matrice triang.
det est le produit elements diagonaux



ou bien

$$= \begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \underset{L_3 \leftarrow L_3 + L_1 + L_2}{=} \begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{vmatrix}$$

L

$$\bullet \quad \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$f(e_1) \quad f(e_2) \quad f(e_3)$

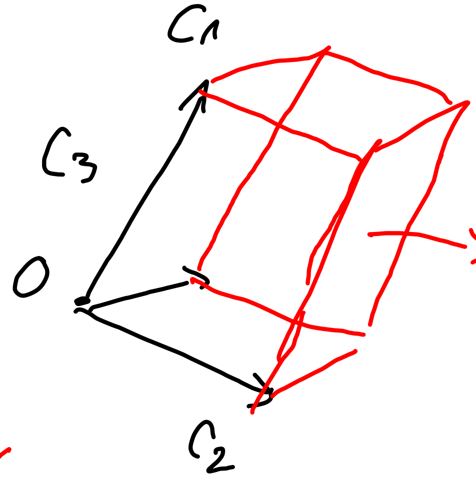
$L_3 \leftarrow L_3 - 3L_1$

$(C_1, C_2, C_1 + C_2)$

$A = (C_1, C_2, C_3)$

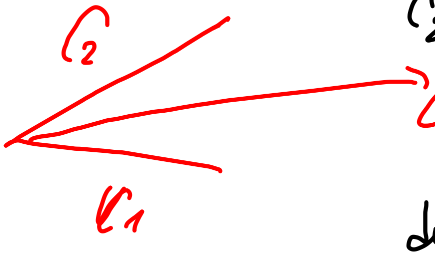
matrice 3×3

$C_2 = 2C_1$



$\rightarrow \det(C_1, C_2, C_2)$

$\overset{\shortparallel}{\text{Vol}}$



$C_3 = C_1\alpha + \beta C_2$

$$\text{mat}_{\mathcal{B}} f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\text{rang}(f) = \dim \operatorname{Im}(f) = \dim \text{Vect}\{f(e_1), f(e_2), f(e_3)\} \overset{\dim}{=} \text{Vect}\{C_1, C_2, \overset{\overset{C_1 + C_2}{\shortparallel}}{C_3}\}$$

$$= \text{Vect}\{C_1, C_2\} \Rightarrow \dim \text{Vect}\, C_1 = 1$$

$$\Rightarrow \dim \operatorname{Ker}(f) = 2$$

$$f(x,y,z) = A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y+3z \\ \cdot \quad - \\ \cdot \quad -\,-\cdot \end{pmatrix}$$

$$f(e_1) \quad f(e_2) \quad f(e_3)$$

$$f(e_2) = 2f(e_1) \longrightarrow f(e_2 - 2e_1) = 0$$

$$f(e_3) = 3f(e_1) \longrightarrow f(e_3 - 3e_1) = 0$$

$$\Longrightarrow e_2 - 2e_1 \in \operatorname{Ker} f$$

$$e_3 - 3e_1 \in \operatorname{Ker} f$$

$$\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \dim \operatorname{Ker} f \geq 2$$

libres

comme $\dim \operatorname{Ker} f = 2$

$$\Rightarrow \operatorname{Ker} f = \operatorname{Vect}\{e_2 - 2e_1, e_3 - 3e_1\}$$

Exercice 2 (F12)

$$D_n = \underbrace{\begin{vmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & 1 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & 1 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & 0 & \vdots & \ddots & \ddots & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 \end{vmatrix}}_{n} \Bigg\updownarrow n$$

$C_2 \leftarrow C_2 + C_1$

$C_3 \leftarrow C_3 + C_1$

$\vdots$

$C_n \leftarrow C_n + C_1$

$$= \begin{vmatrix} 1 & 2 & 2 & 2 & \overset{n}{\cdots} & 2 & 2 \\ \vdots & 2 & 1 & 1 & & & 1 \\ \vdots & 1 & 2 & 1 & \ddots & & \vdots \\ \vdots & \vdots & 1 & 2 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & 1 & \ddots & \ddots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 2 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 2 \end{vmatrix} = \text{On arrête}$$

Nouvelle Tentative :

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 1 & 0 & \cdots & 0 & 1 \end{vmatrix} = ?$$

$$C_1 \leftarrow C_1 - (C_2 + C_3 \cdots + C_n)$$

C_2

$$\underbrace{1 - 1 - 1 \cdots - 1}_{} = 1 - (n-1) = 2 - n$$

$$\begin{vmatrix} 2-n & 1 & 1 & \cdots & 1 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \end{vmatrix}$$

Dvp % C_1

$$= (2-n)\underbrace{(-1)^{1+1}}_{=1} \cdot \begin{vmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{vmatrix} \quad n-1$$

$n-1$

$$\det(I_{n-1}) = 1$$

$$= \underline{\underline{2-n}}$$

Voir une autre preuve par réc. sur le corrigé

D_1

D_2

Ori bine :

$$D_n = \begin{array}{c|ccccc|} & C_1 & & & & \\ & 1 & 1 & 1 & \cdots & 1 \\ & 1 & 1 & 0 & \cdots & 0 \\ & 1 & 3 & \ddots & \ddots & \vdots \\ & \vdots & \vdots & & 1 & 0 \\ & \vdots & 0 & & & 0 \\ L_n & 1 & 0 & \cdots & 0 & 1 \end{array}$$

Ori dup le det %o

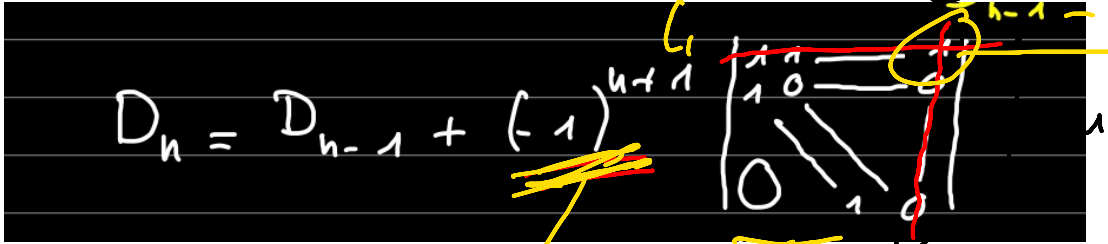
$$L_n = 1\,(-1)^{n+1} \underbrace{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & 0\;1 & 0 \end{vmatrix}}_{n-1} + 1\,\underbrace{(-1)^{n+n}}_{\underbrace{(-1)^{2n}}_{1}} \underbrace{\overbrace{\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 1 & 0 & \cdots & 0 & 1 \end{vmatrix}}^{n-1}}_{D_{n-1}}$$

$$D_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$D_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$D_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$D_n = D_{n-1} + (-1)^{n+1} \underbrace{\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{vmatrix}}_{= ?}$$



$$D_n = D_{n-1} + (-1)^{n+1} \begin{array}{cccc} C_1 & & & C_{n-1} \\ \end{array} \begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 0 & \\ & \ddots & \ddots & & \\ 0 & & & 1 & 0 \end{vmatrix}$$

→ ligne 1 Colonne n-1

↓ au dvp % dernière colonne

$$= D_{n-1} + (-1)^{n+1} \cdot (-1)^{1+n-1} \begin{vmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{vmatrix} \updownarrow n-2$$

$\det(I_{n-2}) = 1$

$$= D_{n-1} + (-1)^{n+1} (-1)^{1+n-1} \cdot 1$$

$$= D_{n-1} + \underbrace{(-1)^{2n+1}}_{-1} = D_{n-1} - 1$$

$$D_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$D_n - D_{n-1} = -1$ Suit. ar. de raison -1

$$D_n = D_2 + (n-2) r = D_2 + (n-2)(-1) = \boxed{2-n}$$