Xavier Fernique (1934–2020)

Xavier Fernique passed away in March 2020. He was one of the world's leading experts in Gaussian random measures and functions, to which he devoted almost all his research, demonstrating fundamental results and making ambitious and fertile conjectures.

After a thesis in Strasbourg (where he spent his entire career) under the supervision[1] of Aimé Fuchs, a student of Robert Fortet, Xavier Fernique very early on became interested in the regularity properties of Gaussian processes, a little away from the major probabilistic trends of the time.

In 1970, in a famous two-page Note to the Comptes Rendus de l'Académie des Sciences de Paris [2] presented by Paul Lévy (for a long time the most cited Comptes Rendus Note in the world – the quotes from MathSciNet are more approximate for the articles before 1997), he proved the strong integrability of the norms of Gaussian random vectors, or suprema of Gaussian processes. The argument, as simple as it is elegant, would have multiple repercussions, until what became known as concentration inequalities [6].

The 1974 course [3] at the "École de Probabilités de Saint-Flour" (for which he was a support of the first editions) on the "Régularité des trajectoires des fonctions aléatoires gaussiennes" is a monument of considerable impact. The pioneering works of Vladimir Sudakov and Richard Dudley on Kolmogorov metric entropy initiated the study of Gaussian processes $(X_t)_{t\in T}$ through the metric properties of the parameter space T equipped with the (pseudo-) distance $\|X_s - X_t\|_2$, $s, t \in T$, of the increments in L^2-norm. Xavier Fernique demonstrates, in this course, that Dudley's sufficient entropy condition is also necessary for the almost sure boundedness and continuity of the trajectories of a stationary Gaussian process. This result will allow definitive progress on random Fourier series, initiated by Jean-Pierre Kahane, with the work of Michael Marcus and Gilles Pisier [5]. This course also develops the notion of "majorizing measure" (on the metric space of parameters), of which Xavier Fernique very early conjectured the relevance to characterize the almost sure continuity of any Gaussian process. This last step will be taken in 1987 by Michel Talagrand [7]. The Fernique-Talagrand theorem is one of the great successes of the second half of the 20th century in the analysis of Gaussian processes (and the interview in the Gazette [9] describes its genesis well). Another result of Xavier Fernique, from the end of the 80s, on random Fourier series with vector coefficients [4] led to the famous conjecture on Bernoulli processes, long promoted by Michel Talagrand and only recently resolved [1] . Michel Talagrand's book [8] (a new edition is to be published) on the "generic chaining", a refined form of majorizing measures (without measure!), is a sum,

[1] with second adviser Pierre Cartier

with multiple facets and ramifications, which fully pays tribute to these innovative and fruitful works.

The teachings and writings of Xavier Fernique entered into the French classical tradition, structured, rigorous, clear and concise. His courses in measure and probability theory gave his young students a taste for these subjects. His scientific lectures were lively and powerful, the tone of his voice often increasing over the minutes. English was not his strong suit, and often foreign colleagues asked him for a presentation in French, which they found more understandable. His tall and robust stature was authoritative. One day, as Stanislaw Kwapień amusedly remembers, he got stuck in the latter's little Fiat 126 Polski, which Western commerce had infiltrated into the Eastern Bloc.

Xavier Fernique's scientific legacy is multiple, from a deep metric understanding of the randomness of Gaussian processes, to Gaussian integrability and concentration, now common properties with somewhat forgotten origins.

In his personal life, he also leaves orphaned a very large family, eight children, many grandchildren and great-grandchildren.

References

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