Admissible shift, reproducing kernel Hilbert space, and abstract Wiener space

The standard Gaussian measure γ_n, with density $\frac{1}{(2\pi)^{\frac{n}{2}}}\, e^{-\frac{1}{2}|x|^2}$, $x \in \mathbb{R}^n$ with respect to the Lebesgue measure on $\mathbb{R}^n$, is not translation invariant. Shifted measures are described by

$$\gamma_n(B+h) \,=\, e^{-\frac{1}{2}|h|^2} \int_B e^{-\langle h,x\rangle} d\gamma_n \tag{1}$$

where $B+h = \{x+h; x \in B\}$, B Borel set in $\mathbb{R}^n$ and $h \in \mathbb{R}^n$. In other words, the shifted measure $\gamma_n(\cdot + h)$ by an element $h \in \mathbb{R}^n$ is absolutely continuous with respect to γ_n, with density $e^{-\frac{1}{2}|h|^2 - \langle h,\cdot\rangle}$.

Let now μ be the Wiener measure on the Borel sets of the Banach space $C([0,1])$ of real continuous functions on $[0,1]$, law of a standard Brownian motion or Wiener process $W = (W(t))_{t\in[0,1]}$. It is not entirely clear to give a meaning to the preceding translation formula in this infinite-dimensional context, and in particular to make sense of $|h|^2$ and $\langle h, \cdot\rangle$. An early result of H. Cameron and W. Martin [7] answers this question in the following form. If (and only if) $h : [0,1] \to \mathbb{R}$ is absolutely continuous on $[0,1]$, with almost everywhere derivative h' in $\mathrm{L}^2([0,1])$ (for the Lebesgue measure), the shifted measure $\mu(\cdot + h)$ is absolutely continuous with respect to μ, with density

$$\exp\bigg(-\frac{1}{2}\int_0^1 h'(t)^2\, dt - \int_0^1 h'(t)dW(t)\bigg),$$

where $\int_0^1 h'(t)dW(t)$ is understood as a Wiener (-Itô) integral.

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This translation formula actually entails some basic features associated to the Wiener measure, namely the so-called Cameron-Martin Hilbert space of absolutely continuous functions on $[0,1]$ with almost everywhere derivative h' in $\mathrm{L}^2([0,1])$, and the Wiener integral $\int_0^1 h'(t)dW(t)$. These objects are in fact only generated by the covariance function of W, $\mathbb{E}(W(s)W(t)) = s \wedge t$, $s,t \in [0,1]$, and give rise to the specific structure consisting of the space $C([0,1])$, with its topology, the Cameron-Martin, or reproducing kernel, Hilbert space, and the Wiener measure.

This structure, called *abstract Wiener space*, may be built for any Gaussian measure (on a Banach space for example), and the text below develops the construction in a rather general setting. While the exposition might appear somewhat abstract, it only relies on some standard functional analysis and is not any longer or difficult than it would be for a specific model like the Wiener space. It covers besides, in a most instructive way, several examples of interest, even finite-dimensional. In addition, it naturally puts forward series representations in orthonormal bases of the reproducing kernel Hilbert space (like the trigonometric or Haar expansions of Brownian motion), a most useful property to transfer, in applications, dimension-free statements from finite to infinite-dimensional Gaussian measures and vectors.

The note is mainly extracted from [12]. Some main expositions on Gaussian measures, vectors, processes, in infinite-dimensional spaces are [16, 4, 11, 13, 14, 8, 10, 5, 18, 19, 15]...

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1 Gaussian measure and random vector

It is classical that the Lebesgue measure λ_n does not extend to an infinite-dimensional setting. However, Gaussian measures, due in particular to their dimension-free features, may easily be considered in infinite-dimensional spaces. A prototype, and central, example is the Wiener measure, with associated Brownian or Wiener process, on the Banach space $C([0,1])$ of continuous functions on the interval $[0,1]$.

A Gaussian measure μ on a real separable Banach space E equipped with its Borel σ-algebra $\mathcal{B}$, and with norm $\|\cdot\|$, is a Borel probability measure on $(E,\mathcal{B})$ such that the law of each continuous linear functional on E is Gaussian. Equivalently, a random variable, or vector, X on some probability space $(\Omega,\mathcal{A},\mathbb{P})$ with values in $(E,\mathcal{B})$ is Gaussian if its law, on the Borel sets of E, is Gaussian, that is, for every element ξ of the dual space E^* of E, $\langle \xi, X\rangle$ is a real Gaussian variable.

By separability of $\mathcal{B}$, the distribution of X may also be described by the finite-dimensional distributions of the random process $\langle \xi, X\rangle$, $\xi \in E^*$, and therefore by the covariance operator

$$\mathbb{E}\big(\langle \xi, X\rangle\langle \zeta, X\rangle\big) \,=\, \int_E \langle \xi, x\rangle\langle \zeta, x\rangle d\mu(x), \quad \xi,\zeta \in E^*$$

(for μ the law of X). As such, all the standard properties of finite-dimensional Gaussian random vectors extend to this infinite-dimensional setting.

The infinite dimensional setting may be extended to locally convex vector spaces [6], but for simplicity, the exposition here is limited to Banach spaces.

Throughout the note, only centered Gaussian measures and vectors are considered, without further notice.

2 Wiener space factorization

Let μ be a Gaussian measure on $(E,\mathcal{B})$. As E is separable, μ is a Radon measure in the sense that, for every $B \in \mathcal{B}$,

$$\mu(B) \,=\, \sup\big\{\mu(K); K \subset B, K \,\text{compact in}\, E\big\}.$$

It is known from the integrability properties of norms of Gaussian random vectors (cf. [1]), that

$$\sigma \,=\, \sup_{\xi\in E^*, \|\xi\|\leq 1} \bigg(\int_E \langle \xi, x\rangle^2 d\mu(x)\bigg)^{1/2} < \infty, \tag{2}$$

and actually

$$\int_E \|x\|^p d\mu(x) \,<\, \infty \quad \text{for every } p > 0. \tag{3}$$

The abstract Wiener space factorization of the Gaussian measure μ on $(E, \mathcal{B})$ is given by

$$E^* \xrightarrow{\ j\ } \mathrm{L}^2(\mu) \xrightarrow{\ j^*\ } E,$$

where j is the injection map from E^* into $\mathrm{L}^2(\mu) = \mathrm{L}^2(E, \mathcal{B}, \mu; \mathbb{R})$ (i.e. $j(\xi) = \langle \xi, \cdot \rangle \in \mathrm{L}^2(\mu)$), the dual map j^* of j mapping $\mathrm{L}^2(\mu)$ into E (rather than the bi-dual). Indeed, by the integrability property (3), for any element φ of $\mathrm{L}^2(\mu)$, the integral $\int_E x\varphi(x) d\mu(x)$ is defined, as an element of E, in the strong sense since

$$\int_E \|x\| \big|\varphi(x)\big| d\mu(x) \,\le\, \bigg(\int_E \|x\|^2 d\mu(x) \bigg)^{1/2} \bigg(\int_E |\varphi|^2 d\mu \bigg)^{1/2} \,<\, \infty.$$

Now, for every $\xi \in E^*$,

$$\langle j(\xi), \varphi \rangle_{\mathrm{L}^2(\mu)} \,=\, \int_E \langle \xi, x \rangle \varphi(x) d\mu(x) \,=\, \bigg\langle \xi, \int_E x\varphi(x) d\mu(x) \bigg\rangle$$

so that $j^*(\varphi) = \int_E x\varphi(x) d\mu(x) \in E$.

3 Reproducing kernel Hilbert space

The reproducing kernel Hilbert space $\mathcal{H}$ of μ is defined as the subspace $j^*(\mathrm{L}^2(\mu))$ of E. By the preceding, its elements are of the form $\int_E x\varphi(x) d\mu(x)$ with $\varphi \in \mathrm{L}^2(\mu)$. This description induces a natural scalar product on $\mathcal{H}$ via the covariance of μ by

$$\langle j^*(\varphi), j^*(\psi) \rangle_{\mathcal{H}} \,=\, \langle \varphi, \psi \rangle_{\mathrm{L}^2(\mu)}, \quad \varphi, \psi \in \mathrm{L}^2(\mu).$$

Since $j(E^*)^\perp = \mathrm{Ker}(j^*)$, j^* restricted to the closure E_2^* of E^* in $\mathrm{L}^2(\mu)$ is linear and bijective onto $\mathcal{H}$. For simplicity in the notation, set below for $h \in \mathcal{H}$,

$$\widetilde{h} \,=\, ({j^*}_{|E_2^*})^{-1}(h) \,\in\, E_2^* \subset \mathrm{L}^2(\mu).$$

Under μ, $\widetilde{h}$ is Gaussian with variance $|h|_{\mathcal{H}}^2$.

Note that σ of (2) is then also $\sup_{x \in \mathcal{K}} \|x\|$ where $\mathcal{K}$ is the closed unit ball of $\mathcal{H}$ for this Hilbert space scalar product. In particular, for every x in $\mathcal{H}$,

$$\|x\| \,\le\, \sigma\, |x|_{\mathcal{H}}$$

where $|x|_{\mathcal{H}} = \langle x, x \rangle_{\mathcal{H}}^{1/2}$. Moreover, $\mathcal{K}$ is a compact subset of E. Indeed, if $(\xi_n)_{n\in\mathbb{N}}$ is a sequence in the unit ball of E^*, there is a subsequence $(\xi_{n'})_{n'\in\mathbb{N}}$ which converges weakly to some ξ in E^*. Now, since the ξ_n's are Gaussian under μ, $\xi_{n'} \to \xi$ in $\mathrm{L}^2(\mu)$ so that j is a compact operator. Hence j^* is also a compact operator, from which the compactness of $\mathcal{K}$ follows.

The terminology "reproducing kernel" stems from the fact that an element $\varphi \in \mathrm{L}^2(\mu)$ is reproduced, by duality, from the covariance kernel of μ as

$$\int_E \varphi\, \psi \, d\mu \, = \, K(\varphi, \psi)$$

where ψ is running through $\mathrm{L}^2(\mu)$. A further illustration of this property in the context of Gaussian processes is provided below.

It is useful to visualize the preceding abstract construction on a number of basic examples.

For γ_n the canonical Gaussian measure on $\mathbb{R}^n$ (equipped with an arbitrary norm), it is plain that $\mathcal{H} = \mathbb{R}^n$ with its Euclidean structure, and $\mathcal{K}$ is the Euclidean (closed) unit ball $B(0,1)$.

If X is a Gaussian vector on $\mathbb{R}^n$ with non-degenerate covariance matrix $\Sigma = M^{\top} M$, the unit ball $\mathcal{K}$ of the reproducing kernel Hilbert space associated to the distribution of X is the ellipsoid $M(B(0,1))$.

An infinite dimensional version of γ_n might consist of an infinite sequence $(Y_n)_{n\in\mathbb{N}}$ of independent standard normal random variables (on some probability space $(\Omega, \mathcal{A}, \mathbb{P})$). This sequence does not belong almost surely to the Hilbert space ℓ^2 of square summable sequences, but as soon as $(a_n)_{n\in\mathbb{N}}$ is a (deterministic) sequence in ℓ^2, the new Gaussian sequence $(a_n Y_n)_{n\in\mathbb{N}}$ belongs to $E = \ell^2$, and its law μ defines an abstract Wiener space $(E, \mathcal{H}, \mu)$ with reproducing kernel Hilbert space $\mathcal{H}$ given by the infinite-dimensional ellipsoid consisting of the sequences $(b_n)_{n\in\mathbb{N}}$ such that $\big(\frac{b_n}{a_n}\big)_{n\in\mathbb{N}}$ belongs to ℓ^2 (assuming the a_n's different from zero).

Another illustrative, infinite-dimensional, example is the classical Wiener space associated with Brownian motion, say on $[0,1]$ and with real values for simplicity (cf. [2]). Let thus E be the Banach space $C([0,1])$ of all real continuous functions x on $[0,1]$ equipped with the uniform norm (the Wiener space), and let μ be the distribution of a standard Brownian motion, or Wiener process, $W = (W(t))_{t\in[0,1]}$ starting at the origin (the Wiener measure). The dual space of $C([0,1])$ is the space of signed measures on $[0,1]$, and if m and m' are finitely supported measures on $[0,1]$, $m = \sum_i c_i \delta_{t_i}$, $c_i \in \mathbb{R}$, $t_i \in [0,1]$, $m' = \sum_j c'_j \delta_{t'_j}$, $c'_j \in \mathbb{R}$,

$t'_j \in [0,1]$,

$$\int_E \langle m,x\rangle\langle m',x\rangle d\mu(x) \;=\; \mathbb{E}\big(\langle m,W\rangle\langle m',W\rangle\big)$$
$$= \sum_{i,j} c_i c'_j \,\mathbb{E}\big(W(t_i)W(t'_j)\big)$$
$$= \sum_{i,j} c_i c'_j (t_i \wedge t'_j)$$

by definition of the covariance of Brownian motion. It follows that the element $h = j^* j(m) = \int_E x\langle m,x\rangle d\mu(x)$ of $\mathcal{H}$ is the map $h : t \in [0,1] \mapsto \sum_i c_i(t_i \wedge t)$. This map is absolutely continuous, with almost everywhere derivative h' satisfying

$$\int_0^1 h'(t)^2\,dt \;=\; \int_0^1 \bigg|\sum_i c_i \mathbb{1}_{[0,t_i]}\bigg|^2 dt$$
$$= \int_0^1 \sum_{i,j} c_i c_j \mathbb{1}_{[0,t_i]}\mathbb{1}_{[0,t_j]} dt$$
$$= \sum_{i,j} c_i c_j (t_i \wedge t_j) \;=\; \int_E \langle m,x\rangle^2 d\mu(x) \;=\; |h|^2_{\mathcal{H}}.$$

By a standard extension, the reproducing kernel Hilbert space $\mathcal{H}$ associated to the Wiener measure μ on E may then be identified with the Cameron-Martin Hilbert space [7] of the absolutely continuous elements h of $C([0,1])$ such that $\int_0^1 h'(t)^2 dt < \infty$. Moreover, if $h \in \mathcal{H}$,

$$\widetilde{h} \;=\; ({j^*}_{|E_2^*})^{-1}(h) \;=\; \int_0^1 h'(t)dW(t)$$

as a Wiener (-Itô) integral, defining a Gaussian random variable with mean zero and variance $\int_0^1 h'(t)^2 dt$.

While the Wiener space $C([0,1])$ is equipped here with the uniform topology, other choices are possible. Let F be a separable Banach space such that the Wiener process W belongs almost surely to F. Using probabilistic notation, the previous abstract Wiener space theory indicates that if φ is a real valued random variable, on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, with $\mathbb{E}(\varphi^2) < \infty$, then $h = \mathbb{E}(W\varphi) \in F$. Since $\mathbb{P}(W \in F \cap C([0,1])) = 1$, it immediately follows that the Cameron-Martin Hilbert space may be identified with a subset of F, and is also the reproducing kernel Hilbert space of the Wiener measure on F. Examples of subspaces F include the Lebesgue spaces $\mathrm{L}^p([0,1])$, $1 \le p < \infty$, or the Hölder spaces with exponent α, $0 < \alpha < \frac{1}{2}$, given by

$$\|x\|_\alpha \;=\; \sup_{0\le s\ne t\le 1} \frac{|x(s)-x(t)|}{|s-t|^\alpha}, \quad x \in C([0,1]).$$

4 Gaussian process

The construction of the reproducing kernel Hilbert space $\mathcal{H}$ of the law of a Gaussian random vector with values in a Banach space may be, at least formally, extended to the setting of Gaussian processes. By definition, a Gaussian process $X = (X_t)_{t\in T}$, on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, indexed by a parameter set T, is a random process such that any finite-dimensional vector $(X_{t_1}, \ldots, X_{t_n})$, $t_1, \ldots, t_n \in T$, is a Gaussian vector in $\mathbb{R}^n$. The finite-dimensional distributions of the process $X = (X_t)_{t\in T}$ are therefore fully determined by the the covariance function $\Sigma(s,t) = \mathbb{E}(X_s X_t)$, $s, t \in T$. As for the Brownian motion, the associated reproducing kernel Hilbert space $\mathcal{H}$ is the span of the functions $s \mapsto \Sigma(s,t)$, $t \in T$, with scalar product

$$\langle h, k\rangle_{\mathcal{H}} = \sum_{i,j} c_i d_j \, \Sigma(s_i, t_j)$$

whenever $h = \sum_i c_i \, \Sigma(s_i, \cdot,)$, for a finite collection of $c_i \in \mathbb{R}$, $s_i \in T$, and similarly $k = \sum_j d_j \, \Sigma(\cdot, t_j)$, and

$$\mathbb{E}\bigg(\Big| \sum_i c_i X_{s_i} \Big|^2 \bigg) = \langle h, h\rangle_{\mathcal{H}}^2 .$$

5 Abstract Wiener space

In the preceding context of a Gaussian measure μ on a Banach space E with reproducing kernel Hilbert space $\mathcal{H}$, the triple

$$(E, \mathcal{H}, \mu)$$

is called, following L. Gross [9], an abstract Wiener space.

A dual point of view, starting from a given Hilbert space, more commonly used by analysts on Wiener spaces, may be emphasized (cf. [11] for further details). Let $\mathcal{H}$ be a real separable Hilbert space with norm $|\cdot|_{\mathcal{H}}$ and let $e_1, e_2, \ldots$ be an orthormal basis of $\mathcal{H}$. Define a simple additive measure ν on the cylinder sets in $\mathcal{H}$ by

$$\nu\big(x \in \mathcal{H}\,; \big(\langle x, e_1\rangle, \ldots, \langle x, e_n\rangle\big) \in B\big) = \gamma_n(B)$$

for all Borel sets B in $\mathbb{R}^n$. Let $\|\cdot\|$ be a measurable semi-norm on $\mathcal{H}$, and denote by E the completion of $\mathcal{H}$ with respect to $\|\cdot\|$. Then $(E, \|\cdot\|)$ is a real separable Banach space. If $\xi \in E^*$, consider $\xi_{|\mathcal{H}} : \mathcal{H} \to \mathbb{R}$ that is identified with an element h in $\mathcal{H} = \mathcal{H}^*$ (in the preceding language, $h = j^* j(\xi)$). Let then μ be the (σ-additive) extension of ν on the Borel sets of E. In particular, the distribution of $\xi \in E^*$ under μ is Gaussian with mean zero and variance $|h|_{\mathcal{H}}^2$. Therefore, μ is a Gaussian Radon measure on E with reproducing kernel Hilbert space $\mathcal{H}$, and $(E, \mathcal{H}, \mu)$ is an abstract Wiener space. With respect to this approach, the abstract Wiener space construction of the preceding sections focuses more on the Gaussian measure.

6 Series representation

The next property is a useful series representation of Gaussian random vectors which can efficiently be used to transfer (dimension-free) properties from finite-dimensional to infinite-dimensional Gaussian measures. The Cameron-Martin translation formula (see the next section) may for example be approached in this way. Another illustration is the extension of the isoperimetric inequality to infinite-dimensional Gaussian measures (cf. [3]).

The result puts besides forward the fundamental Gaussian measurable structure consisting of the canonical Gaussian product measure on $\mathbb{R}^{\mathbb{N}}$ with reproducing kernel Hilbert space ℓ^2.

Theorem 1. *Let $(E, \mathcal{H}, \mu)$ a Wiener triple, $(e_k)_{k\geq 1}$ an orthonormal basis of $\mathcal{H}$, and $(g_k)_{k\geq 1}$ a sequence of independent real standard normal variables on some probability space $(\Omega, \mathcal{A}, \mathbb{P})$. Then the series $X = \sum_{k=1}^{\infty} g_k e_k$ converges in E almost surely and in every L^p, and is distributed according to μ.*

In the example of the Wiener measure on the space $E = C([0,1])$ of continuous functions on $[0,1]$, any orthonormal basis $(h_k)_{k\geq 1}$ of $\mathrm{L}^2([0,1])$ for the Lebesgue measure, gives rise to a Schauder basis

$$e_k(t) = \int_0^t h_k(s)ds, \quad t \in [0,1],\, k \geq 1,$$

of $E = C([0,1])$ to which the preceding Theorem 1 applies. Now, in this concrete example, specific bases $(h_k)_{k\geq 1}$ are of interest, such as the trigonometric or Haar bases. Each of them actually provides a simple approach to continuity of the Brownian paths (cf. [2]).

Theorem 1 actually entails a somewhat more precise statement. Since μ is a Radon measure, the space $\mathrm{L}^2(\mu)$ is separable and the closure E_2^* of E^* in $\mathrm{L}^2(\mu)$ consists of Gaussian random variables on the probability space $(E, \mathcal{B}, \mu)$. Let $(g_k)_{k\geq 1}$ denote an orthonormal basis of E_2^*, and set $e_k = j^*(g_k)$, $k \geq 1$. Then $(e_k)_{k\geq 1}$ defines a complete orthonormal system in $\mathcal{H}$, and $(g_k)_{k\geq 1}$ is a sequence on $(E, \mathcal{B}, \mu)$ of independent standard Gaussian random variables.

A proof of Theorem 1 may, for example, be obtained from a vector valued-martingale convergence theorem (although a direct approach in many specific situations is often easier to apprehend). Here are some details. Recall that $\int_E \|x\|^p d\mu(x) < \infty$ for every $p > 0$. Denote by $\mathcal{B}_n$ the σ-algebra generated by $g_1, \ldots, g_n$. It is easily seen that the conditional expectation of the identity map on (E, μ) with respect to $\mathcal{B}_n$ is equal to $X_n = \sum_{k=1}^{n} g_k e_k$.

By the vector-valued martingale convergence theorem, see [17], the series $X = \sum_{k=1}^\infty g_k e_k$ converges almost surely and in any L^p-space. Since moreover $e_k = \int_E x\varphi_k d\mu$, $k \geq 1$, where $(\varphi_k)_{k\geq 1}$ is an orthonormal basis of $\mathrm{L}^2(\mu)$ (by the reproducing kernel property),

$$\mathbb{E}\big(\langle \xi, X\rangle^2\big) \,=\, \sum_{k=1}^\infty \langle \xi, e_k\rangle^2 \,=\, \sum_{k=1}^\infty \bigg(\int_E \langle \xi, x\rangle \,\varphi_k d\mu\bigg)^2 \,=\, \int_E \langle \xi, x\rangle^2 d\mu(x)$$

for every ξ in E^*, so that X has law μ, and the last claim follows.

As a consequence of this series representation, it may be deduced that the closure $\overline{\mathcal{H}}$ of $\mathcal{H}$ in E coincides with the support of μ (for the topology given by the norm on E), a property that shows the coherence of the abstract Wiener space construction.

7 Cameron-Martin translation formula

After the preceding somewhat lengthy developments, this last section addresses the translation formula for infinite-dimensional Gaussian measures. Actually, the series representation in an orthonormal basis of the reproducing kernel Hilbert space may be used to access the Cameron-Martin translation formula discussed in the introduction from its finite-dimensional version (cf. e.g. [5, 14]).

Theorem 2 (The Cameron Martin formula)**.** *On an abstract Wiener space $(E, \mathcal{H}, \mu)$, for any h in $\mathcal{H}$, the shifted probability measure $\mu(\cdot + h)$ is absolutely continuous with respect to μ, with density given by the formula*

$$\mu(B+h) \,=\, e^{-\frac{1}{2}h|^2_{\mathcal{H}}} \int_B e^{-\widetilde{h}} d\mu \tag{4}$$

for every Borel set B in E, where it is recalled that $\widetilde{h} = ({j^*}_{|E_2^*})^{-1}(h)$.

As developed first in [7], it takes an explicit form on the standard Wiener space. Namely, for $h \in \mathcal{H}$, $\widetilde{h} = ({j^*}_{|E_2^*})^{-1}(h) = \int_0^1 h'(t)dW(t)$, so that if μ is the Wiener measure on $E = C([0,1])$, the shifted measure $\mu(\cdot + h)$ has density

$$\exp\bigg(-\frac{1}{2}\int_0^1 h'(t)^2\, dt - \int_0^1 h'(t)dW(t)\bigg)$$

with respect to μ.

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