Sheaves, schemes, cohomology : an introduction

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Abstract:

The goal of this course is to provide an introduction to the theory of schemes and their cohomologies with quasi-coherent coefficients.

The ultimate goal of this course is the description of the cohomology of projective spaces and its applications to intersection theory (e.g. Bezout formula for the intersection of hypersurfaces).

1. Notions of schemes: Spectrum of prime ideals. Open and closed sets and ideals. Zariski topology. Points and residual fields. Ringed and locally ringed spaces. Schemes. Open and closed sub-schemes, fibered products. Relations with algebraic sets over algebraically closed fields.

2. Sheaves of O-modules: quasi-coherent, coherent and locally free sheaves. Exact sequences and cohomology. Vanishing theorems for cohomology of affine schemes.

3. Cohomology of projective spaces: projective spaces on a ring. Cohomology of line bundles. Application: Grothendieck finiteness theorem.

4. Grothendieck groupe: Grothendieck group of coherent sheaves. Computation of the Grothendieck group of projective spaces. Application: Bezout theorem and Gauss-Bonnet formulas.

References:

1. R. Hartshorne, *Algebraic geometry* , Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977. xvi+496 pp.