Reading Seminar
Regularization of inverse ill-posed problems

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In this seminar, we consider the resolution of linear equations of the form $Tf = g$, where f is the unknown object to be recovered, g is the data, and $T\colon F \to G$ is a compact operator from a Banach space F to a Banach space G, both of them being infinite dimensional.

One faces such problems in a large variety of applications, such as imaging, signal analysis, statistics. Let us mention for example aperture synthesis (in astronomy and space imaging), medical imaging (emission tomography, transmission tomography, MRI, thermoacoustic or photoacoustic tomography, etc.), problems of deconvolution, or nonparametric instrumental regression in statistics.

Most regularization techniques give rise to a variational formulation. It is notably the case of the celebrated Tikhonov regularization, but also of more recent approaches using the L^1-norm, the BV-norm, or the mollification approach. These techniques then rely on a good deal of variational analysis.

The seminar will be organized as follows :

1. Motivations (aperture synthesis, tomography, deconvolution, etc).
2. Classical regularization theory (least squares, generalized inverse, Tikhonov regularization).
3. Modern variational Methods (L^1-norm, BV-norm, approximate inverses, mollification, etc.).
4. Some applications (tomography, instrumental regression).

Prerquisites. Basic functional analysis (Hilbert spaces, operators, least squares, pseudo-inverses, spectral theorem for compact Hermitian operators).

Références

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