Kähler-Einstein metrics on compact Kähler manifolds

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Abstract:

An important problem in Kähler Geometry is to find a Kähler metric on a compact Kähler manifold with "nice properties" that reflect the topological and the differential properties of the manifold. This has been made precise by E. Calabi in the early fifthies in an ambitious program. He stated his famous conjecture and posed the problem of the existence of various kinds of "canonical metrics" on a given compact Kähler manifold.

We will prove the main theorem of S.T. Yau solving the Calabi conjecture and study the problem of the existence of Kähler-Einstein metrics on compact Kähler manifolds. These problems boil down to solving various complex Monge-Ampère equations on compact Kähler manifolds.

We will use the variational approach to solve these equations and then establish a priori estimates to study the regularity of their solutions. This approach will need tools from Complex Differential Geometry, Pluripotential Theory and Partial Differential Equations which will be introduced in the course.

References:

1. T. AUBIN : quations du type Monge-Ampère sur les varits kählériennes compactes. (French) Bull. Sci. Math. (2) 102 (1978), no. 1, 6395.

2. J.-P. DEMAILLY : Complex analytic and differential geometry, https://www-fourier.ujf-grenoble.fr/ demailly/

3. G. TIAN : Canonical metrics in Kähler geometry, Lecture Notes ETH Zrich, Birkhäuser, 2000.

4. S.T. YAU : On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. I. Comm. Pure Appl. Math. 31 (1978), no. 3, 339411.

5. V. GUEDJ, A. ZERIAHI : Degenerate complex Monge-Ampère equations, Tracts in Mathematics, no 26, EMS, 2017.