Numerical analysis of problems involving nonlinear boundary conditions

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In this reading seminar we consider several problems and their approximations by the finite element method [1]. Most of the problems we consider arise in solid mechanics and involve nonlinear conditions on (a part of) the boundary [2]. The two basic problems are the one of Signorini (also called contact problem) and the Coulomb friction and both of them can be either written in a stationary or in a dynamic framework.

From a PDE point of view, only the simple static Signorini problem admits a unique solution. For all the other problems, in particular the friction problems and the dynamic problems, the mathematical analysis is not complete and the question of existence and/or uniqueness remains unsolved. The state of art will be recalled.

We begin with considering the simplest finite element method to approach the static Signorini problem. We study the various techniques to obtain the error bounds coming from the approximation.

We then consider the finite element discretization of the other problems and we study the properties of the finite dimensional problems, in particular the questions of existence, uniqueness, multiplicity of solutions, continuation of solutions, consistency, stability.

[1] P.G. Ciarlet, *The finite element method for elliptic problems*, in Handbook of Numerical Analysis, Volume II, Part 1, eds. P.G. Ciarlet and J.L. Lions, North Holland, (1991), 17–352.

[2] J. Haslinger, I. Hlaváček and J. Nečas, *Numerical methods for unilateral problems in solid mechanics*, in Handbook of Numerical Analysis, Volume IV, Part 2, eds. P.G. Ciarlet and J. L. Lions, North Holland, (1996), 313–485.