Reading seminar : Ginzburg-Landau vortices

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The Ginzburg-Landau theory is a physical model which describes superconducting materials. Ginzburg-Landau vortices are small regions where the superconductivity is destroyed.

To obtain a rigorous mathematical understanding of this phenomenon, Bethuel, Brezis and Hélein introduced and studied a simplified model [1, 2], where a cylindrical superconductor of section $\Omega \subset \mathbb{R}^2$ is described by a map $u\colon \Omega \to \mathbb{C}$ which minimizes the energy

$$E_\varepsilon(u) = \int_\Omega \left(\frac{1}{2}|\nabla u|^2 + \frac{1}{4\varepsilon^2}(|u|^2 - 1)^2 \right),$$

with fixed boundary conditions $u = g$ on $\partial\Omega$.

The aim of this reading seminar is to study the regime where the length scale ε becomes very small, that is :

- identify singular points around which the energy concentrates : the vortices;
- and obtain an asymptotic expansion of the energy depending on the positions of the vortices.

Keywords: Calculus of variations, elliptic partial differential equations.

Pre-requisites: Some familiarity with Sobolev spaces and partial differential equations will be helpful.

References

[1] F. Bethuel, H. Brezis, and F. Hélein. Asymptotics for the minimization of a Ginzburg-Landau functional. *Calc. Var. Partial Differential Equations*, 1(2):123–148, 1993.

[2] F. Bethuel, H. Brezis, and F. Hélein. *Ginzburg-Landau vortices*, volume 13 of *Progress in Nonlinear Differential Equations and their Applications*. Birkhäuser Boston, Inc., Boston, MA, 1994.