Special functions and q-calculus

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Abstract:

The course will be composed of two parts, roughly equal in volume: special functions, applications to q-calculus.

1. *Special functions:* This part is of general interest. We shall concentrate on: elliptic curves and elliptic functions; modular functions; theta functions.

2. *Applications to q-calculus:* This part is devoted to a more specialized subject, but one which interacts with various domains in mathematics and physics. We shall insist on the basic theory of q-difference equations and the phenomenon of q-degeneracy to classical calculus when $q \to 1$. Then, according to time and mood, we shall tackle some historical and exciting examples such as *partition numerorum*, Ramanujan (pseudo-)identities, the q-analog of Stokes phenomenon and q-difference Galois theory.

Prerequisites:

General and linear algebra and analytic functions at L3 level; and the first semester course on Riemann surfaces.

References for the first part of the course:

1. Tom M. Apostol, *Modular Functions and Dirichlet Series in Number Theory*, GTM 41, Springer.

2. Serge Lang, *Elliptic Functions*, GTM 112, Springer.

3. E.T. Whittaker & G.N. Watson, *A Course of Modern Analysis*, Cambridge University Press.

References for the second part of the course:

1. George Gasper and Mizan Rahman, *Basic Hypergeometric Series*, Encyclopedia of Mathematics and its Applications 96, Cambridge University Press.

2. G.H. Hardy, *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*, AMS, Chelsea Publishing Company.

3. Jacques Sauloy, *Analytic Study of q-Difference Equations*, *in* Charlotte Hardouin, Jacques Sauloy, Michael F. Singer, *Galois Theories of Linear Difference Equations: An Introduction*, MSM 211, American Mathematical Society.