Introduction to geometric group theory and 3–manifold topology

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Abstract:

The goal of the course is to study the interplay between geometry, algebra and topology which occurs in geometric group theory. We will be particularly interested in the applications of these ideas to the study of 3–manifolds.

In geometric group theory one studies "nice" actions of groups on geometric spaces in order to relate them to algebraic properties of the group (and topological properties if it is the fundamental group of a manifold). A "nice" action means in the best case a discrete cocompact action but we will see that for some purposes more general actions are useful. The spaces on which groups act are mostly negatively curved spaces in the sense introduced in J. Bertrand's first semester course, for example hyperbolic or CAT(0) spaces. A major aim of this course is to introduce many examples of such spaces and actions, and the main example will be hyperbolic manifolds which are the simplest examples of negatively-curved Riemannian manifolds. The topological part of the course will focus on 3–manifolds and their fundamental groups.

More specifically the topics adressed will be the following:

- First part: generalities
  - Cayley graphs, quasi-isometries, the Milnor–Schwarz lemma and the geometry of a finitely generated group;
  - "Abstract" examples: free groups, free products, surface groups;
  - Fundamental groups of compact manifolds, asphericity.
- Second part: construction of hyperbolic manifolds
  - Geometric constructions via Poincaré's polyhedron theorem, Coxeter groups;
  - Dimension 3: Geometrisation and related constructions;
  - Some arithmetic constructions in higher dimensions;
  - A survey of rigidity results (Mostow–Prasad and Calabi–Weil).
- Cubulation and 3–manifolds (most results in this part will not be completely proven)

- The structure of aspherical 3–manifolds: Thurston–Perelman's geometrisation and Thurston's conjectures;
- Wise's program and Agol's theorem: cubulating hyperbolic groups
- Time permitting we will describe some further examples of 3–manifolds and their properties.

Prerequisites: Some metric geometry and differential geometry (covered by the first semester course), basic group theory, point-set topology. An entry-level knowledge of graph theory is useful but not necessary.

References:

- Brian Bowditch, *A course on geometric group theory*, Memoirs Math. Soc. Japan, `http://www.warwick.ac.uk/~masgak/papers/bhb-ggtcourse.pdf` (the content of the first part of the course will follow the first four sections of these notes quite closely).

- Matthias Aschenbrenner, Stefan Friedl, Henry Wilton, *3-manifold groups*, EMS, `https://arxiv.org/abs/1205.0202` (this is a survey which covers the topics in the third chapter).

A more comprehensive reference on geometric group theory is the recent book by Cornelia Druţu and Michael Kapovich, *Geometric group theory* published by the AMS and available at `http://www.math.ucdavis.edu/~kapovich/EPR/ggt.pdf`, but it contains mostly topics that we will not deal with in this course.