

## Course B4 : Qualitative studies of PDEs: a dynamical systems approach

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- **Main goal:** The aim of this advanced course is to provide an overview of some techniques used in the study of partial differential equations from a dynamical systems point of view.
- **Contents:** We will focus on studying coherent structures, such as periodic patterns and traveling waves in spatially extended systems. The lectures will be illustrated and motivated by a variety of simple model problems, such as the Allen-Cahn or Nagumo equation, reaction-diffusion systems and pattern-formation models such as the Swift-Hohenberg equation. We will learn about techniques to study existence and stability of such coherent structures, including bifurcation theory, center manifold theory, and spatial dynamics. More specifically, there will be two different parts in these lectures.
  1. On periodic patterns
    - presentation of the model (Swift-Hohenberg equation) and main objectives
    - introduction to spatial dynamics and bifurcation theory
    - center manifold theorem (CMT) in finite dimension
    - application of the CMT to the Swift-Hohenberg equation in dimension 1: existence of periodic solutions
    - extension of the CMT to infinite dimension
    - application to the Swift-Hohenberg equation in dimension 2: existence of grain boundaries
  2. On traveling waves
    - presentation of the model (Allen-Cahn equation) and main objectives
    - fixed-point method for existence and uniqueness of solutions
    - existence and uniqueness of monotone traveling waves
    - long time dynamics & stability of traveling waves
- **Prerequisite:** There is no real prerequisite for this advanced course except some basic knowledge on ODEs. However, we (strongly) encourage students to take the course: *A4 - Elliptic PDEs and evolution problems*.
- **References:** We suggest two references:

M. Haragus and G. Iooss, *Local Bifurcations, Center Manifolds, and Normal Forms in Infinite-Dimensional Dynamical Systems*, EDP Sciences, Springer, 2011.

T. Kapitula and K. Promislow, *Spectral and Dynamical Stability of Nonlinear Waves*, Applied Mathematical Sciences, Springer, 2013.
- **Key-words:** PDEs, dynamical systems, center manifolds, periodic patterns, traveling waves.