

M2 Advanced Course Proposal

Second Semester 2019-2020

Dan Popovici

Title: “Deformation Theory of Compact Complex Manifolds”

Abstract: We will first study the basic properties of holomorphic families of compact complex manifolds: local C^∞ triviality (Ehresmann’s Theorem), the Kodaira-Spencer fundamental results (semi-continuity of the Hodge numbers, smooth families of elliptic operators and the smooth variation of the associated Green operators when the dimension of the kernel is constant, the Kodaira-Spencer map) and the Kuranishi family (i.e. the local universal family of deformations of the given complex structure) of a given compact complex manifold. The case of Calabi-Yau manifolds (defined as compact Kähler manifolds whose canonical bundle is trivial) will be highlighted, as will the Bogomolov-Tian-Todorov Theorem stating that the Kuranishi family of any Calabi-Yau manifold is unobstructed.

We will then present a few classical results about the behaviour of various classes of compact complex manifolds under deformations of complex structures. Foremost among these will be the Kodaira-Spencer theorem asserting that if a fibre in a family of compact complex manifolds is Kähler, then all sufficiently nearby fibres are again Kähler.

In the last part of the course, we will present the Demailly-Paun result describing the Kähler cone of an arbitrary compact Kähler manifold, together with a simpler recent proof and the original application to the behaviour of the Kähler cone under deformations of the complex structure.

References.

- [Dem 97] J.-P. Demailly — *Complex Analytic and Algebraic Geometry*—<http://www-fourier.ujf-grenoble.fr/~demailly/books.html>
- [DP04] J.-P. Demailly, M. Paun — *Numerical Characterization of the Kähler Cone of a Compact Kähler Manifold* — Ann. of Math. (2) **159(3)** (2004) 1247-1274.
- K. Kodaira — *Complex Manifolds and Deformations of Complex Structures* — Grundlehren der Math. Wiss. **283**, Springer (1986).
- K. Kodaira, D.C. Spencer — *On Deformations of Complex Analytic Structures, III. Stability Theorems for Complex Structures* — Ann. of Math. **71**, no.1 (1960), 43-76.
- D. Popovici — *Sufficient Bigness Criterion for Differences of Two Nef Classes* — Math. Ann. **364** (2016), 649-655.