Course A4 : Elliptic Partial Differential Equations and Evolution Problems

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Abstract:
The aim of this lecture is to provide a background of methods and techniques for the analysis of elliptic partial differential equations and evolution problems and their numerical approximations.

1. Variational models: elasticity, least action principle, Euler-Lagrange equation.
2. Sobolev spaces in a domain of $\mathbb{R}^n$. Traces. Embeddings. Poincaré inequalities.
3. Elliptic partial differential equations in divergence form:
   (a) Lax-Milgram lemma, H^2-regularity, spectral theory of the Laplace operator, maximum principle.
   (b) Elliptic partial differential equations in divergence form: Galerkin approximations, finite elements methods.
4. Evolution problems:
   (a) semigroups, Hille-Yosida theorem, linear parabolic and dispersive equations, Duhamel's formula.
   (b) Dynamical systems: Stability, Liapunov functional, LaSalle Invariance principle.
   (c) Nonlinear parabolic problems.

Prerequisites:
L^p-spaces. Hilbert spaces. Ordinary differential equations.

References:

1. H. Brezis. *Functional Analysis, Sobolev Spaces and Partial Differential Equations.* Universitext, Springer, 2011.
2. Th. Cazenave and A. Haraux. *Introduction aux problèmes d'évolution semi-linéaires.* Ellipses, 1990.
3. A. Ern and J.L. Guermond. *Eléments finis : théorie, applications, mise en oeuvre.* Springer, 2002.
4. L.C. Evans. *Partial Differential Equations.* Graduate Studies in Mathematics **19**, Amer. Math. Soc., 1998.