Convergence of probability measures, functional limit theorems and applications

The goal of this course is to present the fundamental principles of weak convergence of probability measures in metric spaces, as well as classical functional limit theorems. Among the main topics:

- Tight families of probability measures (theorem of Prokhorov),
- Functional limit theorems and invariance principles (Donsker's theorem),
- Gaussian measures in infinite dimension (Wiener measure),
- Infinitely divisible laws, Lévy-Khinchine theorem, stable laws,

- Basics of large deviation theory in finite and infinite dimension (theorems of Cramer, Gärtner-Ellis, Schilder, Sanov,...).

The course will begin by recalling the notions of convergence for sequences of random vectors and of measures in finite dimension. It may be completed by topics chosen by the instructors (as for example: concentration of measure phenomenon, law of iterated logarithm, extreme laws, attraction domains for stable laws, applications of large deviations to spin systems....)

References

1 R. M. DUDLEY, Real Analysis and Probability.

2. A. DEMBO, O. ZEITOUNI, Large Deviations Techniques and Applications.