Reading Seminar on Riemann Surfaces

June 24, 2019

1. Basic facts on holomorphic and meromorphic functions
2. Manifolds, surfaces and the classification of topological surfaces
3. Riemann surfaces definitions and first facts
4. Maps between Riemann Surfaces and the Hurwitz formula
5. Complex tori
6. Covering spaces and monodromy
7. Integration on Riemann surfaces
8. Divisors and meromorphic functions
9. Projective geometry and algebraic curves
10. Spaces of meromorphic functions and forms associated to a divisor
11. Divisors and maps to projective spaces
12. The Riemann Roch theorem and its first applications
13. Applications of the Riemann Roch Theorem
14. A sketch of proof of the Riemann Roch
15. Abel's Theorem
16. Sheaves and Cech cohomology
17. Algebraic sheaves
18. Invertible sheaves line bundles and H^1

1. Basic facts on holomorphic and meromorphic functions

- The Cauchy-Riemann equation ([4] Chapter 2.3)
- Harmonic Funtions ([4] Chapter 2.5)
- Line integrals and Green's Theorem ([4] Chapter 3.1 and 3.2)
- The mean value property and the maximum principle ([4] Chapter 3.4 and 3.5)
- Complex line integrals and Cauchy's theorem ([4] Chapter 4.3 and 4.4)
- Liouville's, Morera's and Goursat's theorem ([4] Chapter 4.5, 4.6 and 4.7 no proofs)
- Power series expansion of an analytic function ([4] Chapter 5.4 and 5.5)
- The zeros of an analytic function ([4] Chapter 5.7)
- Analytic Continuation ([4] Chapter 5.8)
- Laurent Decomposition and Laurent series expansion ([4] Chapter 6.1)
- Singularities of an analytic function ([4] Chapter 6.2)
- The residue theorem ([4] Chapter 7.1)

2.Manifolds, surfaces and the classification of topological surfaces

- The Implicit function theorem and the local inversion theorem
- Definition of manifold and atlases
- The classification of topological two dimensional orientable, compact surfaces (see [3] Chapter 2.1)
- Definition of the fundamental group
- Universal covering
- The universal covering of a compact orientable surface

3. Riemann surfaces definitions and first facts The main point of this talk is to show that plane curves are examples of Riemann surfaces. See [5]Chapter 1.

4. Maps between Riemann Surfaces and the Hurwitz formula

- Definition of holomorphic and meromorphic functions on a RS
- Examples ([5] Chapter 2.2, except complex tori)
- Holomorphic maps between RS ([5] Chapter 2.3 except complex tori)

- Hurwitz's theorem and the degree ([5] Chapter 2.4)

5. Complex tori (for french speaking only)

- Complex tori ([5] Chapter 2.2 on tori and meromorphic functions on it)
- Maps between complex tori and automorphisms of complex tori ([5] Chapter 3.1, and in particular Proposition 1.12)
- Every torus is a projective cubic ([1] pp 100-104)

6. Covering spaces and monodromy

- More elementary examples of RS: Hyperelliptic curve ([5] Chapter 3.1)
- Coverings and monodromy ([5] Chapter 3.4)
- If a Riemann surface has a holomorphic map of degree 2 onto $\mathbb{P}^1$ then it is a hyperelliptic curve ([5] Proposition 4.11)

7.Integration on Riemann surfaces [5] Chapter 4. Main goal : the Residue Theorem and its applications (Theorem 3.17, [5]).

8. Divisors and meromorphic functions

- Divisors and linear equivalence ([5] Chapter 5.1 and 5.2)

9. Projective geometry and algebraic curves

- Basic Projective Geometry ([5] Chapter 3.5)
- Bezout's theorem ([5] Chapter 5.2)
- Plücker formula ([5] Chapter 5.2)
- Curves with nodes and Plücker formula for this case ([5] page 70).

10. Spaces of meromorphic functions and forms associated to a divisor Main goal : defined $L(D)$ and $L^{(1)}(D)$ and prove that they are finite dimensional. Then state Riemann-Roch theorem and test its statement for known cases.

- Spaces of functions and forms associated to a Divisor ([5] Chapter 5.3)
- The statement of the Riemann-Roch Theorem ([5] Theorem 3.11 Chapter 6)
- Test the statement for divisors on $\mathbb{P}^1$.

11. Divisors and maps to projective spaces

- [5] Chapter 5.4

- Algebraic curves ([5] Chapter 4.1)
- Examples

12. Applications of the Riemann Roch Theorem

- [5] Chapter 7.1
- Definition of the canonical map ([5] Chapter 7.2)

13. A sketch of proof of the Riemann Roch

- Laurent tail divisors and the Mittag-Leffer problem ([5] Chapter 6.2)
- The Riemann-Roch Theorem and Serre duality ([5] Chapter 6.3) (no proof)

14. Abel's theorem

- Homology periods and the Jacobian ([5]Chapter 8.1)
- The Abel-Jacobi map ([5]Chapter 8.2)
- Sketch of proof of necessity in Abel's theorem ([5]Chapter 8.3)
- Sketch of proof of sufficiency in Abel's theorem ([5]Chapter 8.4)
- Picard's group
- Abel's theorem for curves of genus one ([5]Chapter 8.5)

11. Sheaves and Cech cohomology

- Sheaves, presheaves and maps ([5]Chapter 9.1 and 9.2)
- Cech cohomology and examples of computation ([5]Chapter 9.3 and 9.4)

12. Algebraic Sheaves See [5] Chapter 10 (no proofs required)
13. Invertible sheaves, line bundles and H^1 [5]Chapter 11.1, 11.2 and 11.3.

References

[1] Michle Audin. -*Analyse complexe*, http://www-irma.u-strasbg.fr/ maudin/analysecomp.pdf

[2] Cartan Henri. - *Elementary theory of analytic functions of one and several variables.*

[3] Donaldson S. - *Riemann Surfaces.*

[4] Gamelin T.W. - *Complex Analysis.*

[5] Miranda R. - *Algebraic curves and Riemann Surfaces.*