Introduction to Riemannian geometry

Jérôme Bertrand [1]

Abstract:

This course is meant to be an introduction to Riemannian geometry with some emphasis on the metric measure space point of view. The second goal of the course is to introduce the audience to analysis on Riemannian manifolds. If time permits, I will also provide a sketchy introduction to comparison geometry (*i.e.* properties of manifolds or spaces satsifying some "curvature bounds").

More specifically the topics adressed will be the following:

- Differential geometry (Reminder)
  - Manifolds (with boundary) and submanifolds, examples.
  - Vector fields. Tangent bundle.
  - (brief reminder on) differential forms
- Riemannian metrics
  - Riemannian metric
  - Examples. Nash's isometric embedding theorem.
  - Levi-Civita connection
  - Geodesics, exponential map
  - Riemannian manifold viewed as a metric space.
  - Normal coordinates
- Densities and Volume
  - Densities on a Riemannian manifold
  - Volume estimates

[1] bertrand@math.univ-toulouse.fr

- Space forms
  - The sphere
  - The hyperbolic space
- Introduction to curvature [Tool box]
- Variation formula(s)
  - Jacobi fields
  - Gauss lemma
  - Conjugate points
- Introduction to analysis on Riemannian manifolds.
  - Laplace-Beltrami operator
  - Riemannian divergence operator, the divergence formula.
  - Maximum principle
  - The Bochner formula on functions.
- Introduction to comparison geometry (optional)

References:

- S. Gallot, D. Hulin, J. Lafontaine, *Riemannian Geometry*, Springer, third edition, 2004.
- M. Do Carmo *Riemannian geometry*, Birkhäuser, 1992.
- I. Chavel, *Eigenvalues in Riemannian geometry*, Elsevier, 1984.