Course A5 : Convex Analysis / Optimization and Applications

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Abstract:
This course is meant to provide a broad introduction to Convex analysis; numerical optimization; and to study the properties of optimization models commonly used in image processing. The content of the course will include:

1. Elements of convex analysis : Convexity (strict/strong), continuity, lower semi-continuity, functions with a lipschitz gradient, sub-differential. Coercivity. Existence of a minimizer. Optimality conditions.
2. Constrained optimization : Lagrange multipliers, Karush-Kuhn-Tucker Conditions.
3. Duality : Legendre-Fenchel transform. Fenchel-Rockafellar duality.
4. Numerical optimization: gradient descent, Newton and quasi-Newton algorithms. Non-smooth optimization: proximal gradient algorithm, Accelerated proximal gradient algorithm, projected gradient algorithm.
5. Applications in image processing :
   - Compressed sensing: Sparse modeling, algorithms and their guarantees (ℓ^0 and ℓ^1 minimization under Restricted Isometry Property)
   - Dictionary learning: Model, algorithms and their guarantees.

Prerequisites:
Differential calculus, linear algebra, Functional analysis.

References:
1. R.T. Rockafellar, *Convex analysis.* Princeton University Press, 1970.
2. Y. Nesterov, *Introductory lectures on convex optimization : A basic course* Kluwer Academic Publishers, 2004.
3. D. P. Bertsekas, *Nonlinear Programming* Athena Scientific, 2003.
4. M. Elad, *Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing* Springer, 2010.