Course B3 : Hamilton-Jacobi equations from biology

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Abstract: The objective of this course is to present an asymptotic method to study non-local reaction-diffusion equations from evolutionary biology. This asymptotic analysis leads to a class of Hamilton-Jacobi equations with constraint.

The mathematical modeling of the dynamics of phenotypically structured populations under the effect of natural selection and mutations leads to parabolic integro-differential equations. In the limit of small diffusion (mutation) and in long time, the solutions to such equations concentrate as a sum of Dirac masses corresponding to dominant traits. We will present an approach based on Hamilton-Jacobi equations, that has been developed during the last decade, to study such concentration phenomena.

The first part of this course will be based on the derivation of a class of Hamilton-Jacobi equations with constraint from non-local reaction-diffusion equations modeling selection and mutation. In the second part of the course we will focus on the properties of the solutions to these Hamilton-Jacobi equations. In particular, we will show via some examples how this approach can lead to quantitative results in biological applications. To perform such analysis, we will learn about techniques to obtain regularity estimates for the solutions to the reaction-diffusion equations under study. These estimates allow us to perform an asymptotic analysis of such equations. We will also introduce the notion of viscosity solutions for Hamilton-Jacobi equations and we will learn some basic properties of such weak solutions.

Keywords: Non-local reaction-diffusion equations, Hamilton-Jacobi equations, Asymptotic analysis, Maximum principle, Viscosity solutions, Regularity estimates

Prerequisites: We strongly recommend the students to take the course: Elliptic PDEs and evolution problems.

References :

- G. Barles, *An Introduction to the Theory of Viscosity Solutions for First-Order Hamilton-Jacobi Equations and Applications*, Lecture notes and chapter 2 of the book "Hamilton-Jacobi Equations: Approximations, Numerical Analysis and Applications", 2013.
- B. Perthame, *Transport equations in biology–chapter 2*, Frontiers in Mathematics, Birkhäuser Verlag, 2007.
- G. Barles, S. Mirrahimi, and B. Perthame. *Concentration in Lotka-Volterra Parabolic or Integral Equations: A General Convergence Result.* Methods Appl. Anal., 16(3):321–340, 2009.
- S. Mirrahimi and J-M. Roquejoffre. *A class of Hamilton-Jacobi equations with constraint: Uniqueness and constructive approach.* Journal of Differential Equations, 260(5):4717–4738, 2016.