Introduction to Lévy processes

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Abstract

This course is an introduction to Lévy processes. We will present the class of infinitely divisible distributions and the celebrated Lévy-Khintchine formula. We will then touch on some Markovian feature of Lévy processes. Finally, we highlight the main differences in stochastic calculus with respect to Lévy processes. The prerequisite on this course is the first semester course of stochastic calculus for Brownian motion.

The Brownian motion is the only process that has continuous sample path, independent and stationary increments. Removing the continuity of the sample path, we see that the Poisson process also has independent and stationary increments. Although very different those two processes are part of the larger class of Lévy processes, processes with independent and stationary increments.

In the first part of this course, largely inspired from Bertoin [2], we will establish the celebrated Lévy-Khintchine formula, which characterises the Fourier transform of any Lévy process. We will study the building blocks of the Lévy processes, namely the Brownian motion, the compound Poisson process and the small jumps part averaging to an L^2 martingale, to get to the so called Lévy-Itô décomposition.

In the second part of this course, we will touch on some Markov feature of those processes. In addition to the aforementioned book by Bertoin, we will also be relying on Sato [4] and Protter [3] for this part. We will discuss how the jump part of Lévy processes give rise to a non-local operator in the infinitesimal generator of a Lévy process.

Finally, we will cover the main differences for stochastic calculus when dealing with jumps. We will see that the celebrated Itô's formula still holds, but has to be modified to take into account the jumps of the process. For that last part, we will mainly use Applebaum [1].

References

[1] D. Applebaum. *Lévy Processes and Stochastic Calculus, II Edition.* Cambridge University Press, 2009.

[2] Jean Bertoin. *Lévy Processes.* Cambridge University Press, October 1998.

[3] P. E. Protter. *Stochastic Integration and Differential Equations.* Springer, 2005.

[4] K. Sato. *Lévy processes and Infinitely divisible Distributions.* Cambridge University Press, 2005.

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