Reading seminar C2 : PDEs and applications

Abstract:
This reading seminar will be divided into three independent parts. Each of them will deal with a particular topic aiming at illustrating/developping the basic courses or at preparing the advanced courses of the second semester.

- **Part 1 : The finite element method for non-coercive PDEs**
  Coordinator : Pierre Cantin - `Pierre.Cantin@math.univ-toulouse.fr`

  In this seminar, we consider the extension of the finite element method applied to non-coercive problems. Theses problems are very common in physics and efficient algorithms to approximate physical solutions are essential. We will start this seminar by understanding why the standard finite element method is not optimal for approximating such problems. Then, we will consider two classes of approximation techniques: the residual-based stabilization and the fluctuation-based stabilization. These methods will be applied to various physical problems, such as the advection-reaction problem, the porous media model or the Maxwell system.

  The results seen within the reading seminar will be useful when dealing with the Basic Course A4 and A6.

  References.

  A. Ern and J.-L. Guermond. *Theory and Practice of Finite Elements*, Applied Mathematical Sciences, Springer, 2004.

- **Part 2 : PDE models in biology**
  Coordinator : Grégory Faye - `Gregory.Faye@math.univ-toulouse.fr`

  In this reading seminar, we will present some classes of PDEs that naturally arise in biology (in evolution theory, cell motion and chemotaxis, etc...) together with mathematical techniques to study them. The class of PDEs encountered will range from integral equations, transport PDEs to kinetic equations and their parabolic limits. Along the way, we will introduce concepts such as Hamilton-Jacobi equation, generalized relative entropy, blow-up, diffusive and hydrodynamic limits.

  The results seen within the reading seminar are complementary with the materials of the Basic Courses A4 and A6, and will be useful when dealing with the Advanced Course B3.

  References.

  B. Perthame. *Transport Equations in Biology*, Frontiers in Mathematics, Birkhäuser, 2007.

- **Part 3 : Variational Methods**
  Coordinator : Stefan Le Coz - `stefan.le-coz@math.univ-toulouse.fr`

  Variational methods have proven to be one of the major tools developped for the analysis of nonlinear PDE. In this reading seminar, we will review some of the most common variational techniques for PDE : extrema of functionals on Banach spaces, constrained critical points, deformation lemmas and the Palais-Smale condition, saddle points, min-max methods, the mountain pass theorem. Every item will be illustrated by examples and applications.

  The results seen within the reading seminar are complementary with the material of Basic Course A4 (Elliptic Partial Differential Equations and Evolution Problems).

  References.

  Ambrosetti, Antonio; Malchiodi, Andrea; Nonlinear analysis and semilinear elliptic problems. Cambridge Studies in Advanced Mathematics, 104. Cambridge University Press, Cambridge, 2007. xii+316 pp. ISBN: 978-0-521-86320-9; 0-521-86320-1