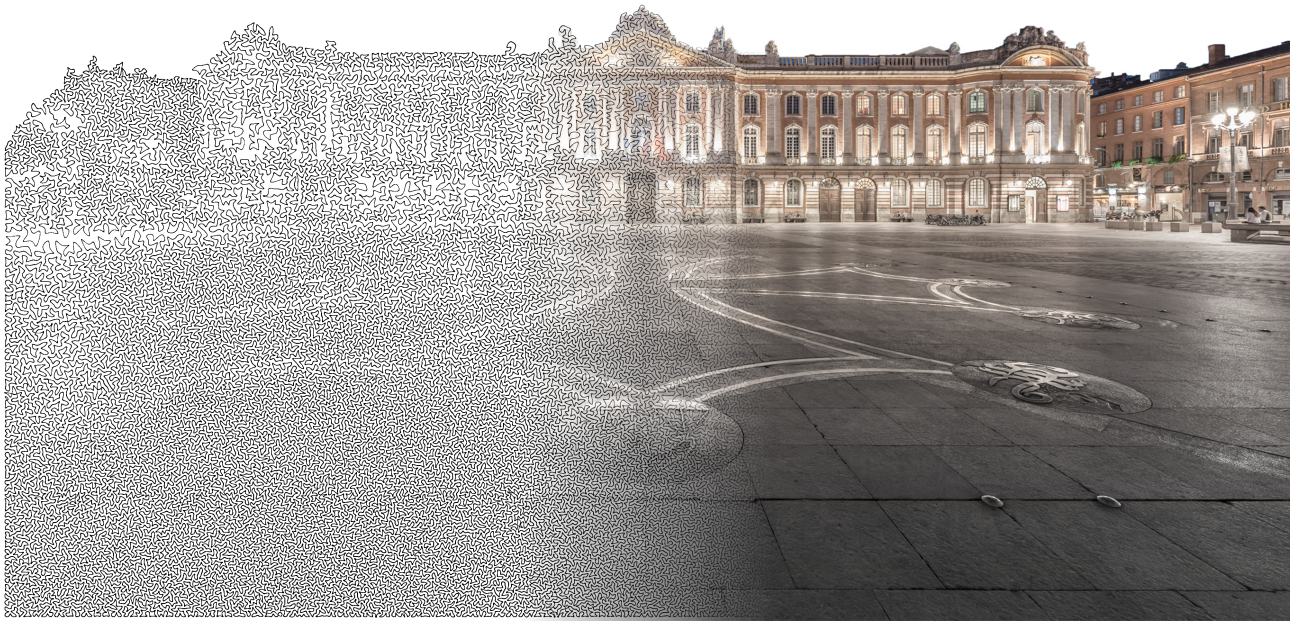
Conference

Optimization on Measures Spaces

Insitut de Mathématiques de Toulouse

25-27th of November 2019



Practical information

Venue

All the conference will take place at the Institut de Mathématiques de Toulouse, 118 route de Narbonne, Toulouse. It is located around 200 meters from the metro line B station Université Paul Sabatier. The speakers will have access to a large blackboard for their presentation.

Receipts

For the invited speakers, please keep all your receipts to get refunded.

Hotel for invited speakers

The invited speakers will be hosted at Aparthotel Adagio Saint-Cyprien, which is located 37 Ter, Avenue Etienne Billieres, 31300 Toulouse. The phone number is +33 5 34 50 96 00,

The fastest and simplest way to reach the Institut de Mathématiques de Toulouse from there is to take the metro A at station Patte d'Oie towards Balma-Gramont, stop at station Jean Jaurès. Then take the metro B, towards Ramonville-Saint-Agne and stop at Université Paul Sabatier.

Lunches

All invited speakers and participants who indicated they would participate to the lunches will eat at l'Esplanade, which is about 200 meters from the Amphithéatre Schwartz.

Gala dinner

The gala dinner will take place on Tuesday, at Péniche Samsara, 1 bvd Montplaisir, on the canal du midi. The boat is booked from 19:30 to 23:30. There will be a unique vegetarian menu with some wine, but it is possible to bring meat and wine on your own. The organizers spotted that many participants are musicians. Do not hesitate to bring your instruments since the place is only dedicated to the conference and is is not a conventional place.

To get there from the university, it will take 30 minutes : you can walk along the canal for 1500 meters starting from station Saouzelong on metro line B. If the distance is too long, you can also take bus 44 and stop at cours Dillon, and walk for 500 meters to reach the boat. To get there from the Aparthotel Adagio, you can take line A and stop at Esquirol and then take bus line 7 towards Saint-Orens de Gameville and stop at arrêt Demouilles.

Troubles

If you have specific needs, do not hesitate to contact the organizers :

Pierre Weiss: pierre.armand.weiss@gmail.com, +33666481185

Jonas Kahn: jonas.kahn@math.univ-toulouse.fr, +33669061434

Program

	Monday 11/25	**Tuesday 11/26**	**Wednesday 11/27**
9:30-10:30	B. Maury	N. Keriven	K. Bredies
10:30-11:00	*Pause*	*Pause*	*Pause*
11:00-12:00	G. Steidl	A. Eftekhari	S. Lisini
12:00-14:00	*Lunch/Posters/Discussions*	*Lunch/Posters/Discussions*	*Lunch/Posters/Discussions*
14:00-15:00	V. Duval	Y. Traonmilin	M. Korda
15:00-16:00	J. Fageot	A. Flinth	G. Bouchitté
16:00-16:30	*Pause*	*Pause*	*Pause*
16:30-17:30	G. Plonka	L. Chizat	

List of abstracts

Guy Bouchitté (bouchitte@univ-tln.fr)

Relaxed multi-marginal costs and quantization effects

(joint work with G. Buttazzo, T.Champion, L. De Pascale)

We propose a duality theory for multi-marginal repulsive cost that appear in optimal transport problems and in Density Functional Theory (quantum chemistry). The related optimization problems involve probabilities on the entire space and, as minimizing sequences may lose mass at infinity, it is natural to expect relaxed solutions which are sub-probabilities. We first characterize the N-marginals relaxed cost in terms of a stratification formula which takes into account all k interactions with k<=N. We then develop a duality framework involving continuous functions vanishing at infinity and deduce primal-dual necessary and sufficient optimality conditions. Next we prove the existence and the regularity of an optimal dual potential under very mild assumptions. In a last part of the talk, we apply our results to a minimization problem involving a given continuous potential and we give evidence of a mass quantization effect for optimal sub-probabilities.

Kristian Bredies (kristian.bredies@uni-graz.at)

Dynamic inverse problems in spaces of measures with optimal-transport regularization

(joint work with Marcello Carioni, Silvio Fanzon and Francisco Romero)

We discuss the solution of dynamic inverse problems in which for each time point, a time-dependent linear forward operator mapping the space of measures to a time-dependent Hilbert space has to be inverted. These problems are regularized with dynamic optimal-transport energies that base on the continuity equation as well as convex functionals of Benamou-Brenier-type. Well-posedness of respective Tikhonov minimization is discussed in detail. Further, for the purpose of deriving properties of the solutions as well as numerical algorithms, we present sparsity results for general inverse problems that are connected with the extremal points of the Benamou-Brenier energy subject to the continuity equation. For the latter, it is proven that the extremal points are realized by point masses moving along curves with Sobolev regularity. This result will be employed in numerical optimization algorithms of generalized conditional gradient type. We present instances of this algorithm that are tailored towards dynamic inverse problems associated with point tracking. Finally, the application and numerical performance of the method is demonstrated for sparse dynamic superresolution.

[1] Kristian Bredies, Silvio Fanzon. An optimal transport approach for solving dynamic inverse problems in spaces of measures. arXiv:1901.10162, 2019.

[2] Kristian Bredies, Marcello Carioni. Sparsity of solutions for variational inverse problems with finite-dimensional data. arXiv:1809.05045, to appear in Calculus of Variations and Partial Differential Equations, 2019.

[3] Kristian Bredies, Marcello Carioni, Silvio Fanzon, Francisco Romero. On the extremal points of the ball of the Benamou-Brenier energy. ArXiv:1907.11589, 2019.

Lénaïc Chizat (lenaic.csl@gmail.com)

Sparse Optimization on Measures with Over-parameterized Gradient Descent

Minimizing a convex function of a measure is a typical problem arising, e.g., in sparse spikes deconvolution or two-layer neural networks training. We show that this problem can be solved by discretizing the measure and running non-convex gradient descent on the positions and weights of the particles (this corresponds to the usual way to train neural networks). For measures on a d-dimensional manifold and with a sparsity-inducing penalty, this leads to a global optimization algorithm with a complexity scaling as log(1/ϵ) in the desired accuracy ϵ, instead of ϵ^d for convex methods. The key theoretical tools are a convergence analysis in Wasserstein space and an analysis of a perturbed mirror descent in the space of measures.

Vincent Duval (vincent.duval@inria.fr)

A representer theorem for convex optimization
(joint work with Claire Boyer, Antonin Chambolle, Yohann De Castro, Frédéric de Gournay and Pierre Weiss)

I will discuss a general principle which describes the solutions of convex variational problems as a sum of a few ``atoms''. That principle has already appeared in different forms in the literature. It is a fundamental tool for the practical resolution of infinite-dimensional variational problems such as the recovery of Radon measures, in conjunction with conditional gradient algorithms. I will show how that principle relates to a theorem by Dubins and Klee in the study of convex sets.

Armin Eftekhari (armin.eftekhari@gmail.com)

Nonnegative Sparse Super-Resolution: Simplified and Stabilized

(joint work with Bogdan Toader, Jared Tanner, Hemant Tyagi, Andrew Thompson, Tamir Bendory, and Gongguo Tang.)

This talk is about the (grid-free) super-resolution problem of localizing positive point sources from (noisy) measurements. Starting with the one-dimensional problem and in the absence of noise, we

observe that the measurement map is injective and that super-resolution is thus possible by solving a convex but infinite-dimensional feasibility program, provided that the measurement functions form a Chebyshev system, namely, a system of continuous functions that loosely behave like polynomials. There is no need here for minimum separation between the sources and the measurement map only needs to be continuous.

This feasibility program is also robust against noise and model mismatch. This technique also generalizes to resolving point sources in (two-dimensional) images. Lastly, super-resolution and many other grid-free inference problems lead to infinite-dimensional optimization programs. In this context, we identify a natural connection between the "sparseland" and semi-infinite programming: The conditional gradient algorithm (CGA) and the exchange method (EM), each popular in one of these fields, are in fact equivalent. This observation helps us develop new convergence guarantees for EM and might lead to new and improved variants of CGA in the future.

Julien Fageot (julienfageot@fas.harvard.edu)

Native Banach Spaces for TV-based Inverse Problems

In this talk, we propose a systematic construction of the native Banach spaces for optimization problems whose regularization involves a Banach space norm and a differential operator L. In short, the native space is the largest space of functions f such that the norm of Lf is finite. The main challenge for the construction is to understand the compatibility between the Banach space and the operator, in particular when the latter has a non-trivial null space. This construction paves the way for the analysis TV-based inverse problems.

Axel Flinth

Linear convergence of an exchange algorithm for super-resolution.

(joint work with F. de Gournay and P. Weiss)

The TV-minimization approach to super-resolution is an SDP, which can be resolved using an exchange algorithm approach. Recent papers have shown that when a natural one-point exchange rule is applied, one in fact ends up with the conditional gradient method. From this, one can readily extract convergence guarantees from the conditional gradient theory. These are however, since they apply much more generally than to our sparse recovery problem, quite weak.

In this talk, we show that under a specialized condition, known as the (in the signal processing community well-known) non-degenerate source condition, a slight modification of the approach outlined above will converge linearly towards a solution of the TV-problem. We will discuss our proof, the strengths and weaknesses of the method, and, if time allows, possible modifications of the algorithm and amendments to the analysis.

Nicolas Keriven (Nicolas.Keriven@ens.fr)

Fisher metric, support stability and optimal number of measurements in compressive off-the-grid recovery

(joint work with Clarice Poon and Gabriel Peyré)

Many problems in machine learning and imaging can be framed as an infinite dimensional Lasso problem to estimate a sparse measure, using randomized projections. In this work, we show that in many cases, the Fisher-Rao distance induced by the measurement process is a natural way to enforce the classical minimal separation condition. We then prove that a number of measurements proportional to the sparsity is enough to obtain recovery guarantees, and that, under additional hypothesis, exact support stability holds when the level of noise is smaller than a specified value.

Milan Korda (milan.korda@laas.fr)

Measure optimization in dynamical systems and control: overview and perspectives

This talk will give an overview of the applications of measure optimization in the fields of dynamical systems and control. We will explain how a number of problems such as optimal control, region of attraction or invariant set computation can be cast as linear programming problems in the space of Borel measures. We will briefly describe how these linear programs can be approximated using the moment-sum-of-squares hierarchy of semidefinite programming problems and how feasible solutions to the dual problem provide certificates of various properties of the dynamical system studied. The talk will finish by discussing how the approach can be used in a data-driven setting where the underlying dynamics is unknown and only observed data are available.

Stefano Lisini (stefano.lisini@unipv.it)

Optimal control problems for interaction equations: Lagrangian and Eulerian formulations and finite particles approximation.

(joint work G. Cavagnari, M. Fornasier, C. Orrieri, G. Savaré)

In this talk I will describe a family of optimal control problems for interaction equations. Equivalence between Lagrangian and Eulerian formulation of the problems will be discussed. A Gamma-convergence result will be illustrated. Techniques of optimization and optimal transportation of measures are used.

Bertrand Maury (Bertrand.Maury@math.u-psud.fr)

Projections in the Wasserstein space

Handling congestion in crowd / cell motion models makes it necessary to properly define the projection of any measure on the subset of measures with a density bounded by a prescribed value, from the optimal transportation standpoint. We shall address some theoretical and numerical issues related to this apparently simple problem. In particular, we shall investigate the similarities and differences of this projection problem with its discrete counterpart, i.e. the problem which consists in projecting a union of finite size particles possibly overlapping to the feasible set of configurations with no overlapping.

Gerlind Plonka (plonka@math.uni-goettingen.de)

Reconstruction of non-stationary signals by the generalized Prony method.

(joint work with T. Peter, K. Stampfer, and I. Keller)

In this talk, we reconsider the problem of parameter identification in short exponential sums which can be solved by the well-known Prony method. The exponential sum can be also interpreted as a sparse linear combination of eigenfunctions of the shift operator. This view led to a generalization of Prony's method in Peter & Plonka (2013), where we have shown that sparse expansions of eigenfunctions of linear operators can be reconstructed completely by using only a small number of suitable sample values. In this talk, we consider special classes of generalized shift operators and corresponding sets of eigenfunctions that admit a reconstruction of structured functions from function values. In particular, we can show that the reconstruction of expansions of shifted Gaussians, Gabor expansions with Gaussian window functions, Gaussians with different scaling as well as non-stationary signals with special monotone phase functions can be reconstructed by the generalized Prony method.

Gabriele Steidl

Curve Based Approximation of Measures on Manifolds by Discrepancy Minimization

(joint work with M. Ehler, M. Gräf and S. Neumayer)

The approximation of probability measures on compact metric spaces and in particular on Riemannian manifolds by atomic or empirical ones is a classical task in approximation and complexity theory with a wide range of applications. Instead of point measures we are concerned with the approximation by measures supported on Lipschitz curves. Special attention is paid to push-forward measures of Lebesgue measures on the interval by such curves. Using the discrepancy as distance between measures, we prove optimal approximation rates in terms of Lipschitz constants of curves. Having established the theoretical convergence rates, we are interested in the numerical minimization of the discrepancy between a given probability measure and the set of push-forward measures of Lebesgue measures on the interval by Lipschitz curves. We present numerical examples for measures on the 2- and 3-dimensional torus, the 2-sphere, the rotation group in 3D and the Grassmannian of all 2-dimensional linear subspaces of the four-dimensional Euclidean space.

Our algorithm of choice is a conjugate gradient method on these manifolds which incorporates second-order information. For efficiently computing the gradients and the Hessians within the algorithm, we approximate the given measures by truncated Fourier series and use fast Fourier transform techniques on these manifolds.

Yann Traonmilin (yann.traonmilin@u-bordeaux.fr)

Projected gradient descent for non-convex sparse spike estimation

(joint work with J.-F. Aujol and A. Leclaire)

We propose an algorithm to perform sparse spike estimation from Fourier measurements. Based on theoretical results on non-convex optimization techniques for off-the-grid sparse spike estimation, we present a simple projected descent algorithm coupled with an initialization procedure. Our algorithm permits to estimate the positions of large numbers of Dirac masses in 2d from Fourier measurements. This opens the way for practical estimation of such signals for imaging applications as the algorithm scales well with respect to the dimensions of the problem. We present, along with the algorithm, theoretical qualitative insights explaining its success.

References:

- Yann Traonmilin, Jean-François Aujol, Arthur Leclaire. Projected gradient descent for non-convex sparse spike estimation. 2019. URL: https://hal.archives-ouvertes.fr/hal-02311624v1

- Yann Traonmilin, Jean-François Aujol. The basins of attraction of the global minimizers of the non-convex sparse spike estimation problem. 2019. URL: https://hal.archives-ouvertes.fr/hal-01938239