Mini-course: Shape spaces

Alain Trouvé

Abstract

As the modern analysis is intimately linked with the definition and properties of various functional spaces, the modern analysis of population of shapes should rely on adequate shapes spaces. Driven mainly by applications to the analysis of the variability of biological shapes, the mini-course will be a gentle introduction to some of the key aspects of the theory, developed in the last fifteen years, that considers shape spaces as homogeneous spaces under large group of transformations, mainly group of diffeomorphisms of the ambient space.

The mini-course will be splitted into four chapters and will hopefully follow the following structure:

1. Introduction to shape spaces
   First examples. Group action and D'Arcy Thompson point of view and homogeneous spaces. Riemannian manifolds. Equivariante distances and metric on quotient spaces.
2. Linear and non-linear matchings and the LDDMM framework
   Reproducible Hilbert Spaces and linear interpolation. Admissible Hilbert strucure on vector fields and associated group of diffeomorphisms. Induced riemaniann metric and existence of geodesics and variational formulation of the Large Deformation Diffeomorphic Mapping (LDDMM) point of view.
3. Geodesic flows, hamiltonian dynamics and extensions
   Optimal control point of view. Geodesics as Hamiltonian flows. Shooting methods and computational issues (how to do it fast with no pain with Keops). Karcher means. Parallel transport.
4. Geometrical data and loss
   Invariance to reparametrisation, discretizations and data attachments for geometric data. Measures, currents, varifolds. Optimal transport. New numerical framework. The Geomloss package.