Cartan method of moving frames revisited

Alice Barbara Tumpach

Abstract

Recall that the space of parameterized curves on a Lie group or an homogeneous space is fibered over the shape space of unparameterized curves. In this talk, we consider the problem of finding canonical sections of this fiber bundle. For curves in the Euclidian space, an example of canonical section is given by curves parameterized by arclength, or, if one choose the parameter space to be [0,1], proportional to arclength. Canonical sections are related to the following question : suppose that you have only 50 points to represent a complicated smooth curve, where should you distribute your points? Parameterization by arclength will lead to equidistributed points along the curve, and will blur the details of the curve. In order to minimize loss of information, one may prefer the parameterization proportional to curvature-length, where points distribute according to the curvature of the curve. We will show that this problem is related to the method of moving frames introduce by Elie Cartan in 1936, and to the theory of invariant of curves and surfaces.