Incorporating knowledge in large deformations

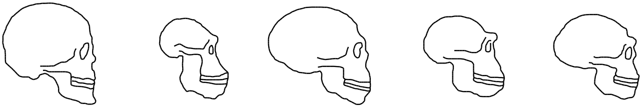
Barbara Gris

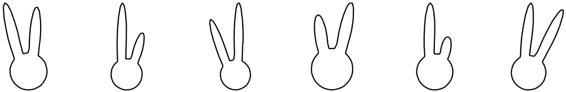
bgris.maths@gmail.com
LJLL, UPMC, Paris

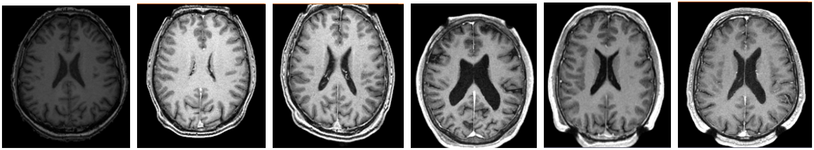
Joint work with Benjamin Charlier (Université de Montpellier), Stanley Durrleman (ICM, Paris), Leander Lacroix (UPMC, Paris), Alain Trouvé (ENS Paris-Saclay)

INTRODUCTION

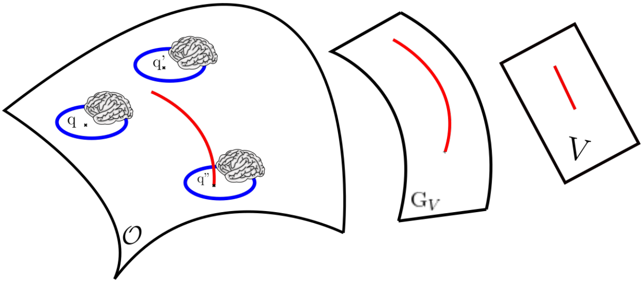
Studying populations of shapes



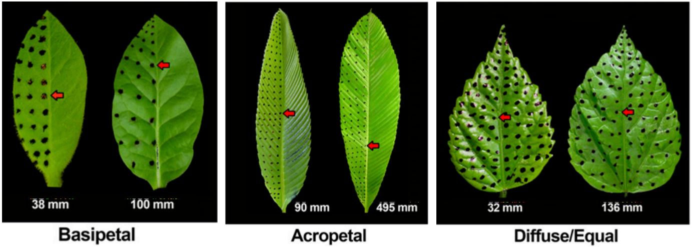




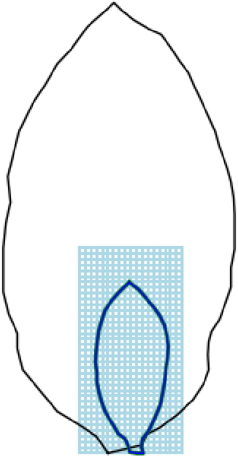
Large deformation diffeomorphic metric mapping



$\longrightarrow$ Talks of Alain Trouvé, Martin Bauer.

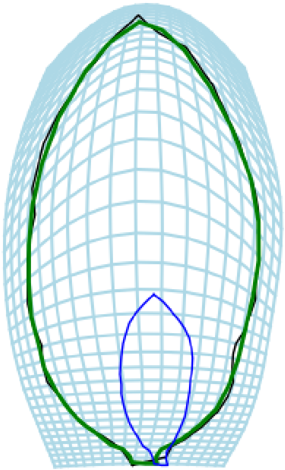


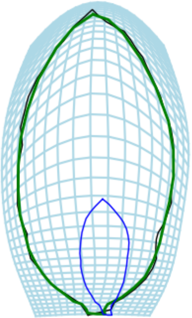
[Gupta, M. D., Nath, U. (2015). Divergence in patterns of leaf growth polarity is associated with the expression divergence of miR396. The Plant Cell, tpc-15.]

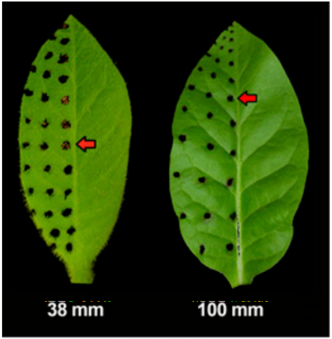


$\longrightarrow$ Talks of Alain Trouvé

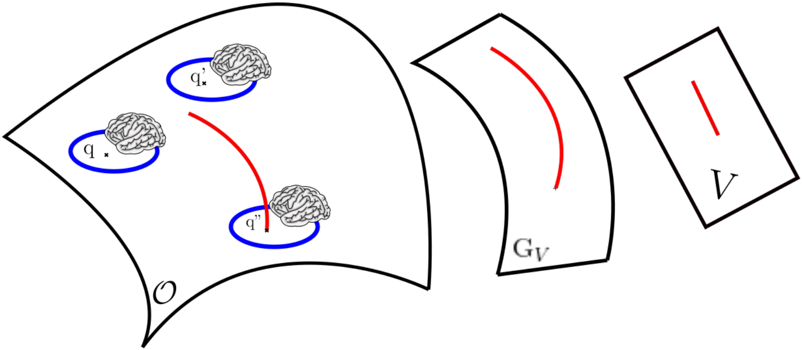
Non parametric







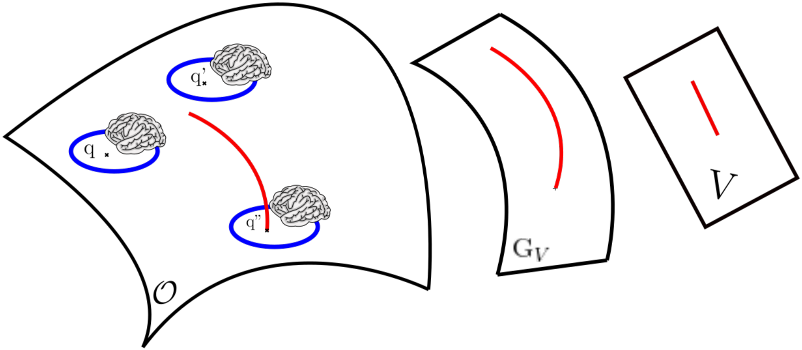
Basipetal

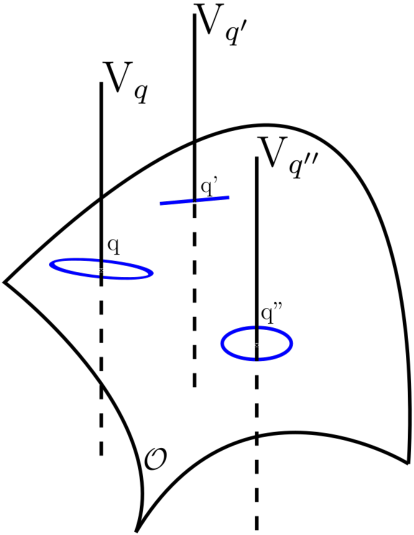

q'
q
q''
$\mathcal{O}$
G_V
V

Incorporating a structure in large deformations:

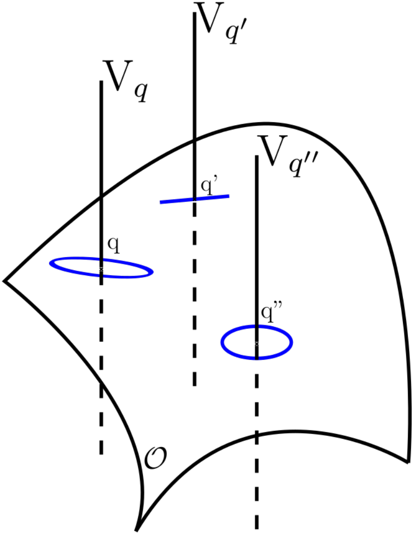
- Sparse LDDMM (Deformetrica) [S. Durrleman, M. Prastawa, G. Gerig, and S. Joshi. Optimal data-driven sparse parameterization of diffeomorphisms for population analysis. In Information Processing in Medical Imaging , pages 123-134. Springer, 2011]
- Higher order momentum [S. Sommer M. Nielsen, F. Lauze, and X. Pennec. Higher-order momentum distributions and locally affine lddmm registration. SIAM Journal on Imaging Sciences, 2013]
- GRID [U. Grenander , A. Srivastava , S. Saini. A pattern-theoric characerization of biological growth. IEEE, 2007]
- Poly-affine [V. Arsigny, X. Pennec, N. Ayache, 2005. Polyrigid and Polyaffine Transformations: A Novel Geometrical Tool to Deal with Non-rigid Deformations – Application to the Registration of Histological Slices. Medical Image Analysis 9, 507–523]
- Diffeons [L. Younes. Constrained diffeomorphic shape evolution. Foundations of Computational Mathematics, 2012.]
- Elastic LDDMM [Hsieh, D. N., Arguillère, S., Charon, N., Miller, M. I., Younes, L. A Model for Elastic Evolution on Foliated Shapes. 2018]

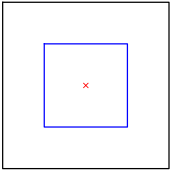
DEFORMATION MODULE


q'
q
q"
$\mathcal{O}$
G_V
V

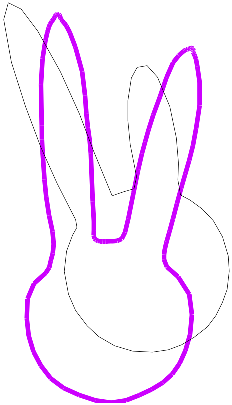


- Trajectories:
  - $\dot{q}_t = v \cdot q_t$, $v \in V_{q_t}$
  - Length: $\int_0^1 |\dot{q}_t|_{V_{q_t} \cdot q_t} \mathrm{d}t$
- Model:
  - Field generator $q \mapsto V_q = \zeta_q(H)$
  - Metric $|\dot{q}|^2_{V_q \cdot q} \doteq \inf\{c_q(h) \mid \dot{q} = \zeta_q(h)\}$
- Optimal control: $\inf \int_0^1 c_{q_t}(h_t)\mathrm{d}t + g(q_{t=1})$ with $\dot{q}_t = \zeta_{q_t}(h_t) \cdot q_t$.
  - Existence of minimizer
  - Geodesic shooting (Hamiltonian)

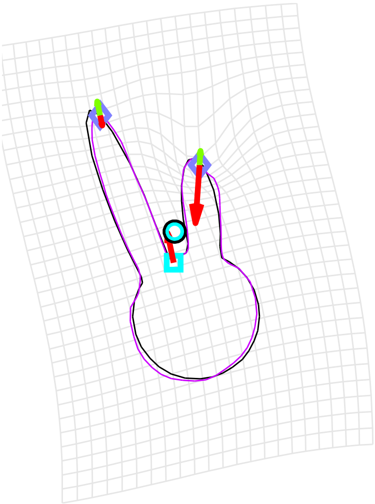


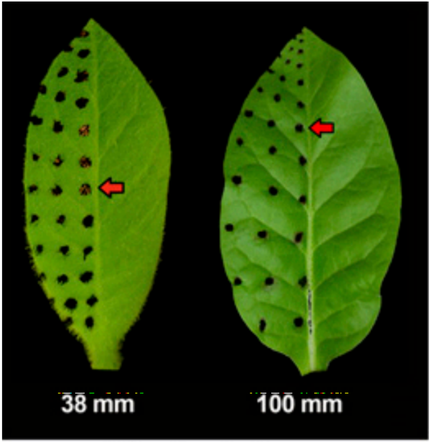


- Extended shape space $\tilde{q} = (q, \theta)$
- $V_{\tilde{q}} = V_\theta$
- $\dot{\tilde{q}}_t = (v \cdot q_t, v \cdot \theta_t), \; v \in V_{\tilde{q}}$
- Combination:
  - $\tilde{q} = (q, \theta, \psi)$
  - $V_{\tilde{q}} = V_\theta + V_\psi$
  - $\dot{\tilde{q}}_t = (v \cdot q_t, v \cdot \theta_t, v \cdot \psi_t), \; v \in V_{\tilde{q}}$



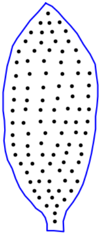
t=1





Basipetal

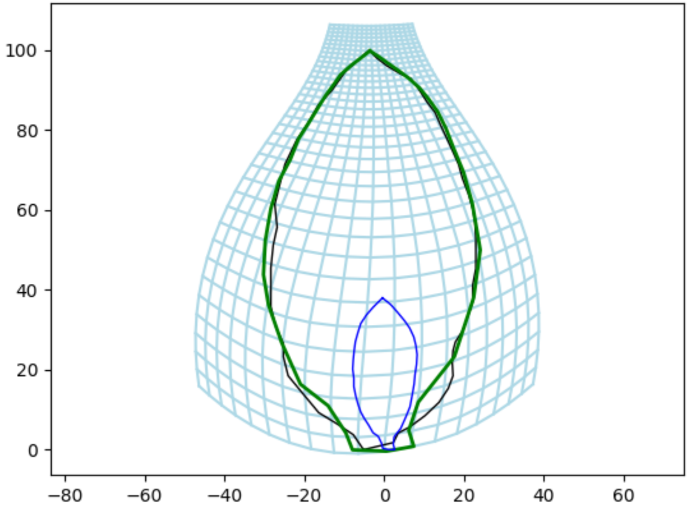
IMPLICIT DEFORMATION MODULES

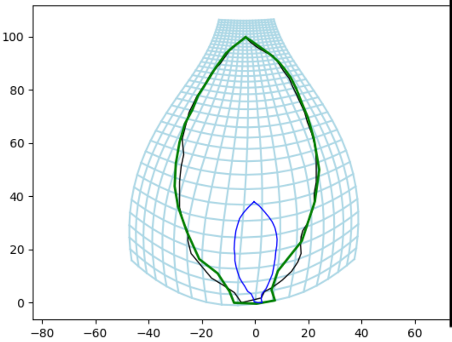


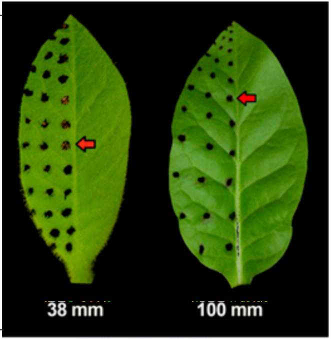
- Shape q
- $\theta = (x_i)_{1 \leq i \leq N}$
- $V_\theta = \{v \in V \mid Cons_\theta(v, h) = 0\}$
  $\longrightarrow$ Implicit module
- With $\epsilon_{x_i}(v) = \frac{Dv(x_i) + Dv(x_i)^T}{2}$, $h \in \mathbb{R}$

$$C_\theta(v, h) = \sum_i |\epsilon_{x_i}(v) - hS_i|^2$$

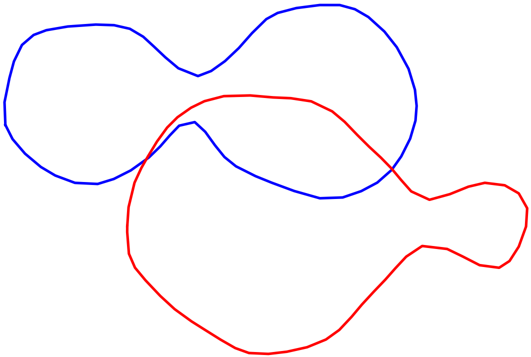
- $\zeta_{\tilde{q}}(h) = argmin\{Cons_\theta(v, h) + |v|_V^2\}$

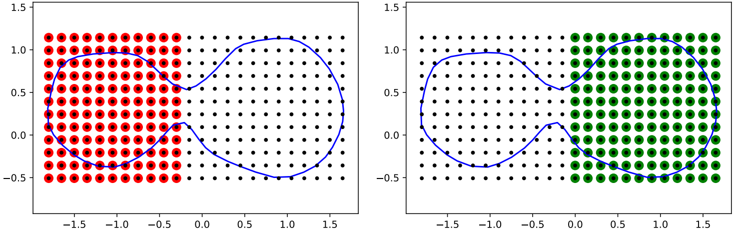

100
80
60
40
20
0
−80
−60
−40
−20
0
20
40
60



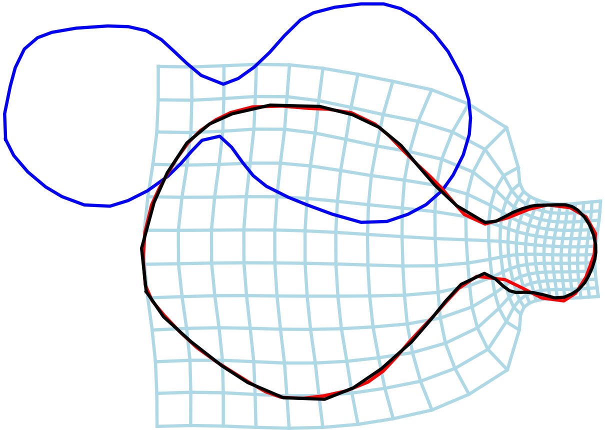


Basipetal

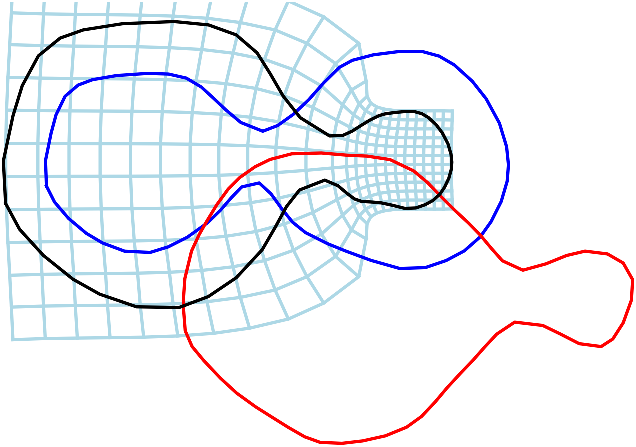


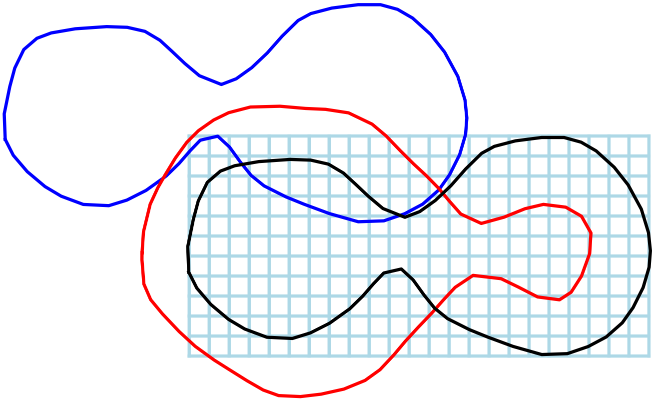


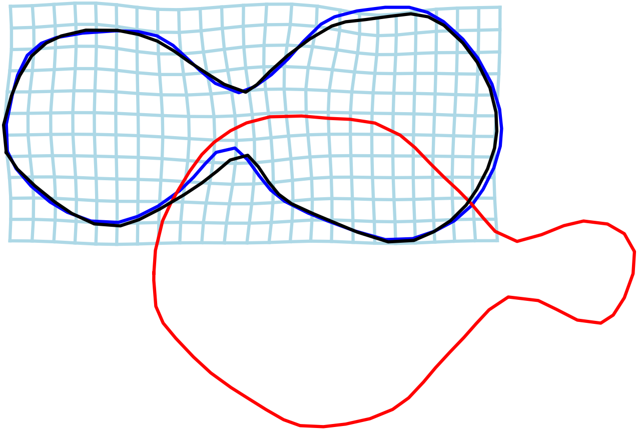
- Shape q
- $\theta = (x_i)_{1 \leq i \leq N}$
- $V_\theta = \{v \in V \mid \frac{Dv(x_i)+Dv(x_i)^T}{2} = (h_1 \delta_{x_i \in I_1} + h_2 \delta_{x_i \in I_2}) Id\}$

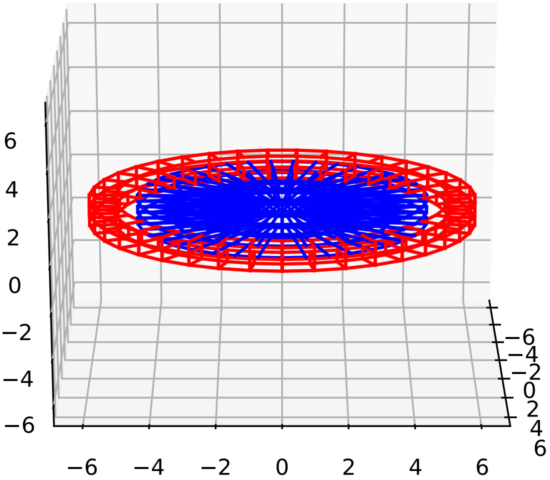


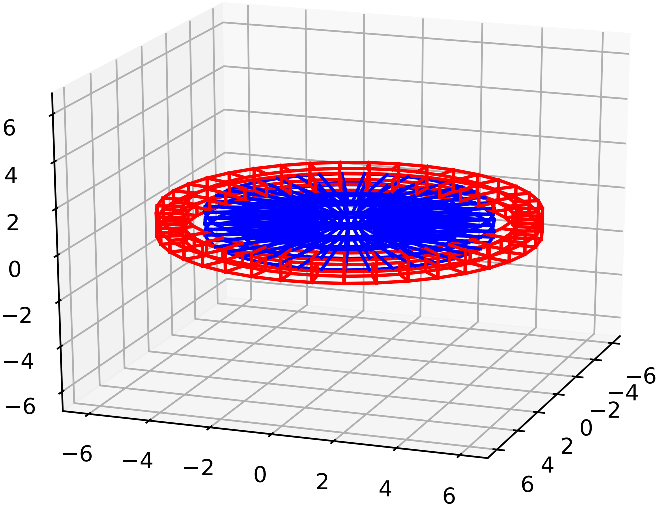
- Combination of 3 types of deformations
- Possibility to follow one of them

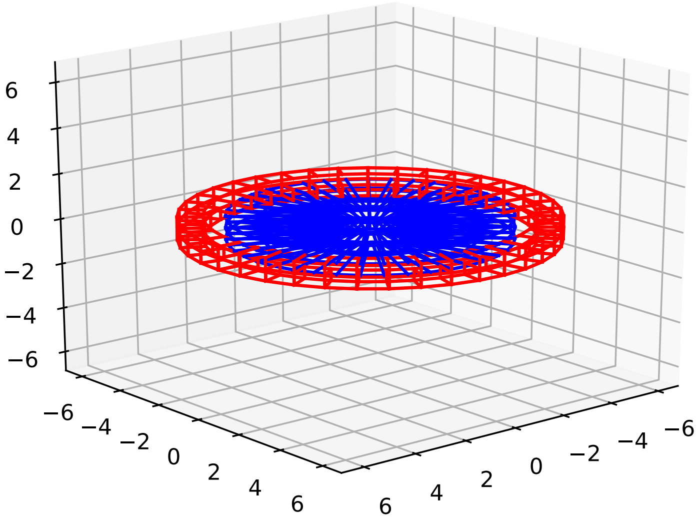


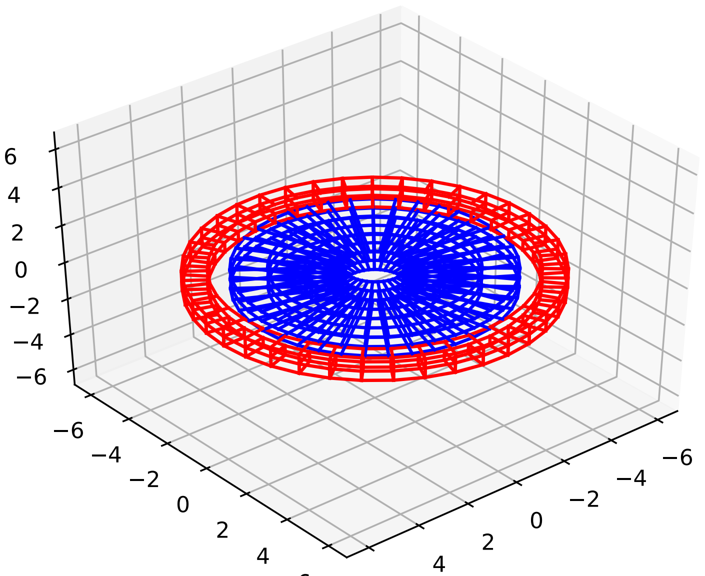


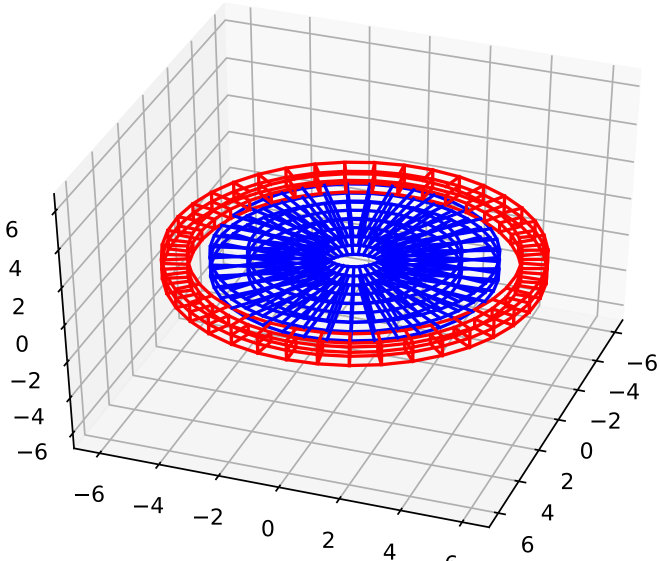


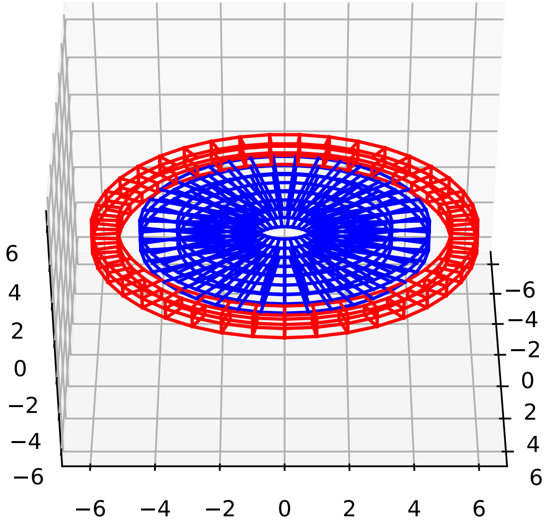

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

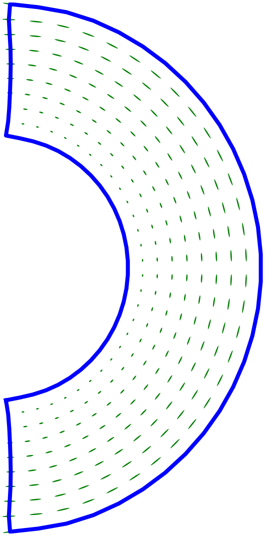

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
−6
−4
−2
0
2
4

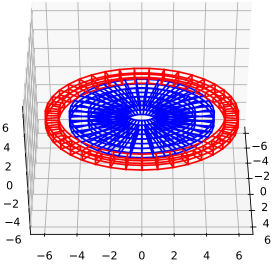

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
−6
−4
−2
0
2
4
6

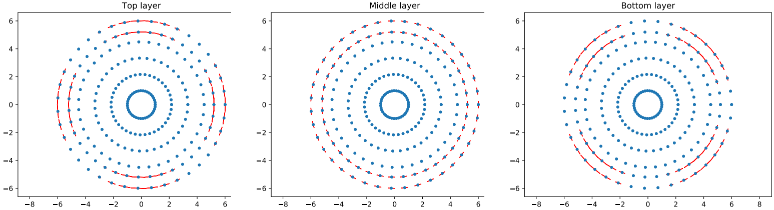

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

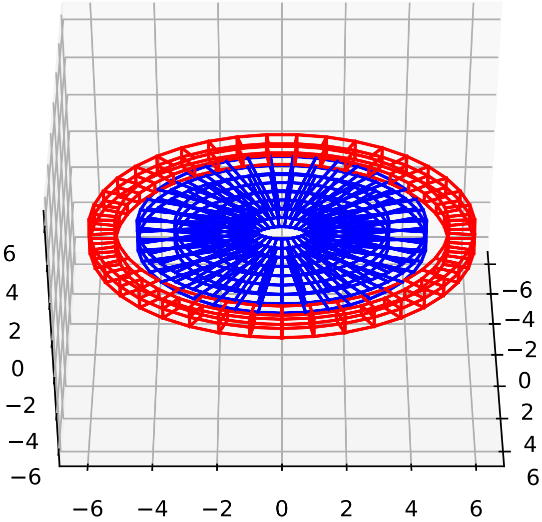


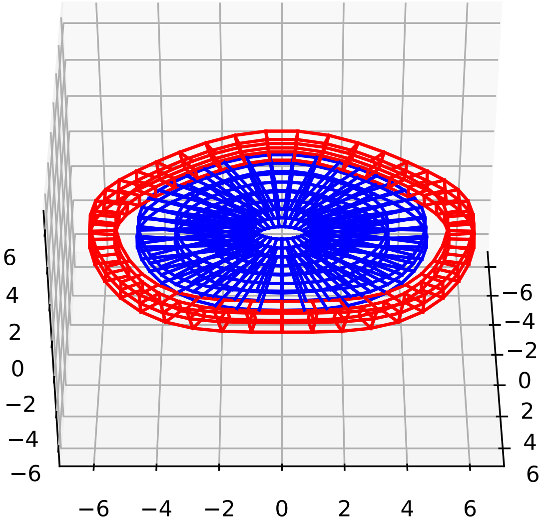
$$R_{\theta_i}(t) = \varphi_t \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

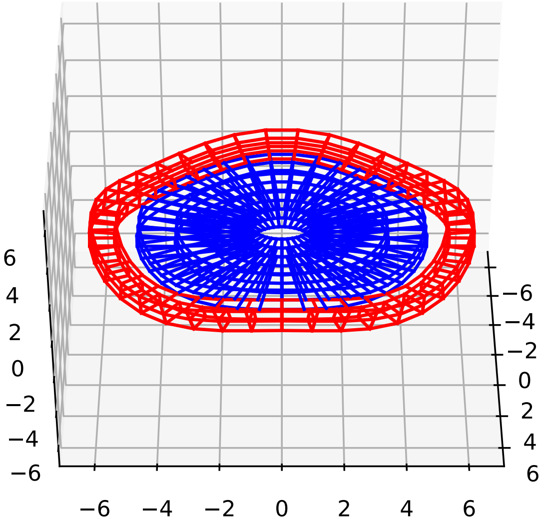
$$Cons_\theta(v, h) = h \begin{pmatrix} 0 & 0 \\ 0 & x_i[0] - \min x[0] \end{pmatrix}$$

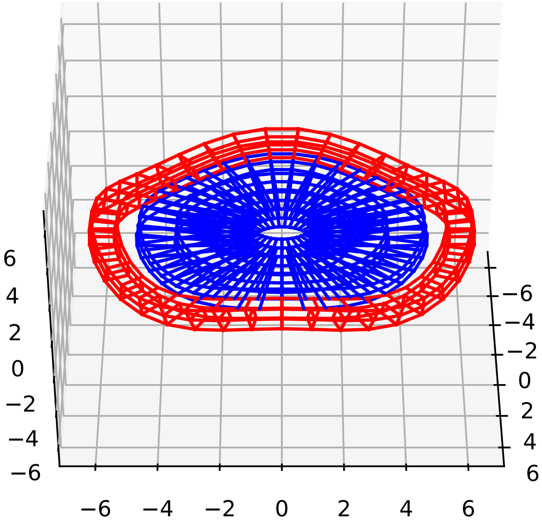


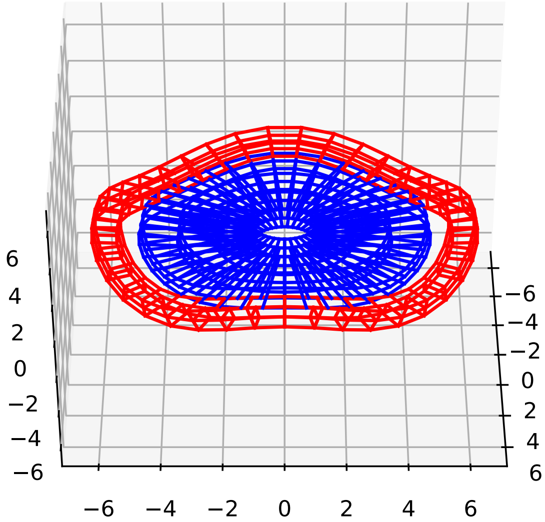

Top layer
Middle layer
Bottom layer

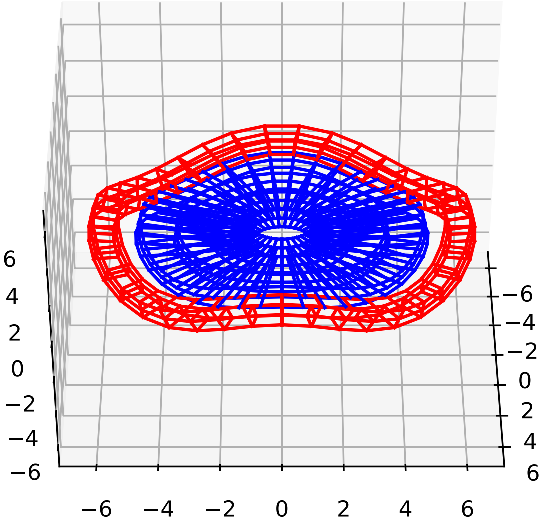

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

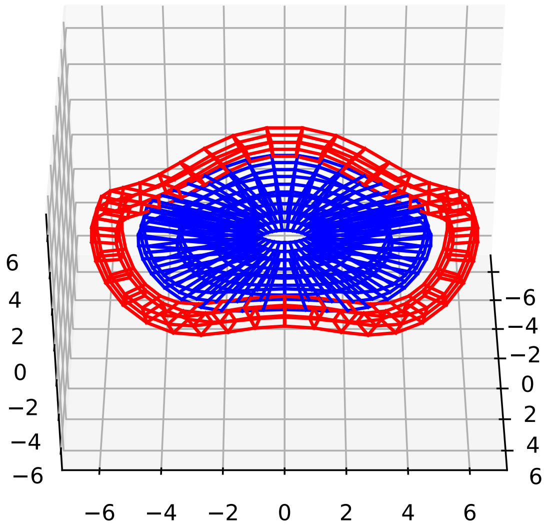

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

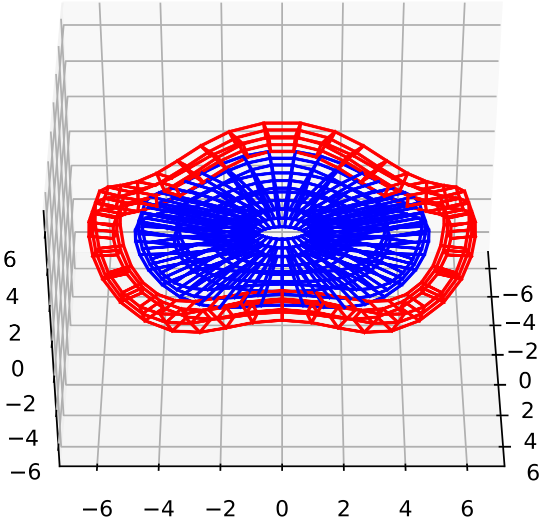

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

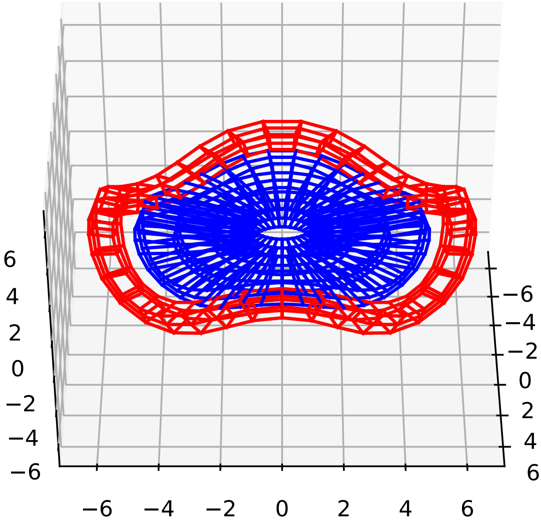

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

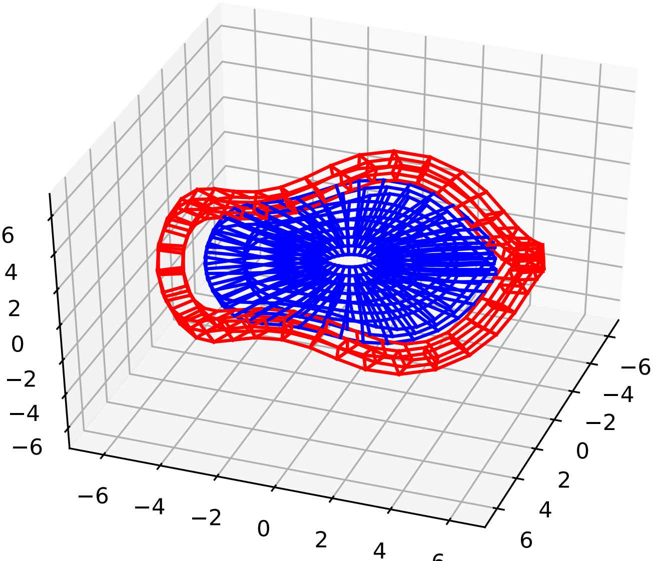

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

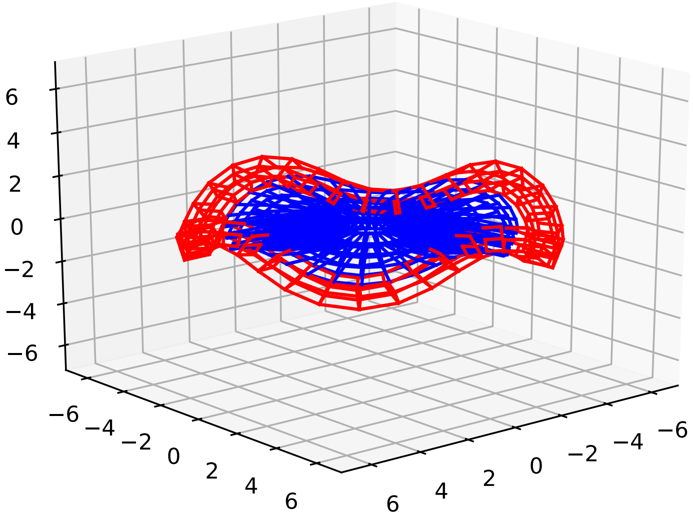

6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6

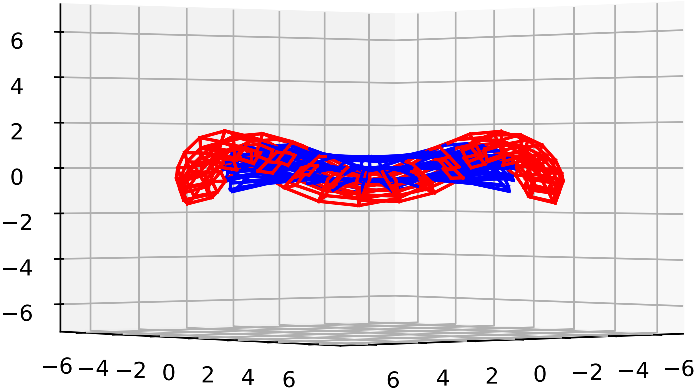

6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6
−6
−4
−2
0
2
4
6

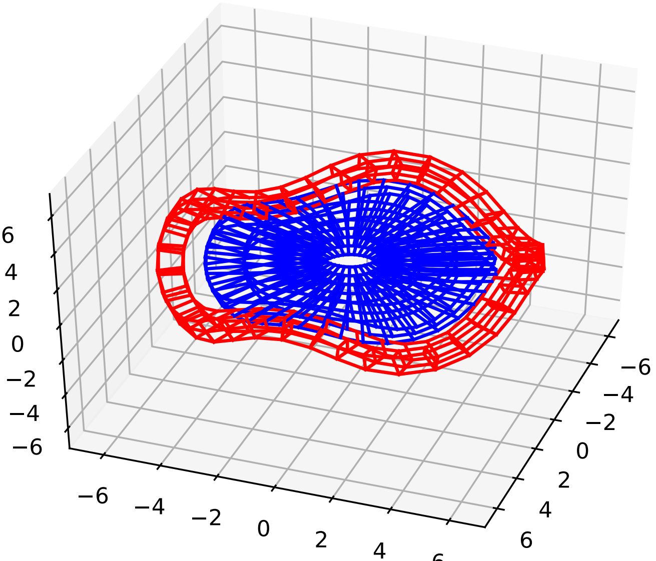

−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6
6
4
2
0
−2
−4
−6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4
6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
−6
−4
−2
0
2
4


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6
−6 −4 −2 0 2 4 6


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6
6 4 2 0 −2 −4 −6


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
−6
−4
−2
0
2
4
6

CONCLUSION

- Incorporating structures in deformations
- Implicit deformation modules:
  - Structure from a biophysical model
  - Learning growth rate
- Study populations (Frechet means)
- Algorithm soon available
- KeOps https://www.kernel-operations.io (B. Charlier, J. Feydy, J. Glaunes)

Workshop *Shape analysis in biology*
21-22 November 2019, Paris

Questions ?