Brown measures and non-normal random matrices

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AKEMANN Gernot: Talk. Universal Local Statistics of Lyapunov exponents
We consider the product of M complex Ginibre random matrices of sizes $N \times N$, being a simple toy model for chaotic dynamical systems.
While the behaviour of the Lyapunov exponents of the product matrix was known in the limit $M \to \infty$ at finite N, taking deterministic values, recent progress has been made at finite M and N. The corresponding exponents follow a determinantal point process and thus all correlation functions of the squared singular values of the product matrix are known. This has allowed us to take a double scaling limit for $N, M \to \infty$, where the Lyapunov exponents exhibits a rich variety of correlations, interpolating between deterministic behaviour and the sine- or Airy-kernel of a single random matrix, depending on the location in the spectrum. Surprisingly, in the bulk we find the same limiting interpolating kernel as for Dyson's Brownian motion for certain initial conditions as constructed by Johansson.
This is joint work with Zdzislaw Burda and Mario Kieburg `https://arxiv.org/abs/1809.05905` [math-ph].

BLITVIC Natasha: Talk. "Type B Gaussian Statistics and Noncommutative Central Limits"
We will show how the noncommutative Gaussian statistics associated with Coxeter groups of type B, as introduced by Bożejko, Ejsmont, and Hasebe, arise as limits in a (noncommutative) Central Limit Theorem. We will discuss some of the general motivation behind these questions, as well as what the underlying ideas may tell us more broadly. Based on recent joint work with Wiktor Ejsmont.

BORDENAVE Charles: Talk. "Spectral gap of sparse bistochastic matrices with exchangeable rows with application to shuffle-and-fold maps". This is a joint work with Yanqi Qiu and Yiwei Zhang.
We consider a random bistochastic matrix of size n of the form M Q where M is a uniformly distributed permutation matrix and Q is a given bistochastic matrix. Under mild sparsity and regularity assumptions on Q, we prove that the second largest eigenvalue of MQ is essentially bounded by the normalized Hilbert-Schmidt norm of Q when n grows large. We apply this result to random walks on random regular digraphs and to shuffle-and-fold maps of the unit interval popularized in fluid mixing protocols.

CHAFAI Djalil: Mini-course. "Non-normal random matrices. "
This mini-course will be centered around the circular law theorem, which states that the empirical spectral distribution of a non random matrix with i.i.d. entries of variance 1/n

tends to the uniform law on the unit disc of the complex plane as the dimension n tends to infinity.
This phenomenon is the non-Hermitian counterpart of the semi circular limit for Wigner random Hermitian matrices, and the quarter circular limit for Marchenko-Pastur random covariance matrices. We will consider the Gaussian case, which goes back to Ginibre and Mehta, and the universal case which was studied notably by Girko, and by Tao and Vu. We will also try to give a glimpse over related and recent problems, and a list of open problems.
We will roughly follow the scheme of `https://arxiv.org/abs/1109.3343`.

GUIONNET Alice: Talk. TBA.

LEHNER Franz: Talk. "Some examples of Brown measures in Free Probability"
This talk is based on a joint paper with P.Biane from 2001.

NAJIM Jamal: Talk. "Non-hermitian Large Random matrices with a variance profile"
Let $V_n = (\sigma_{ij}^2)$ be a $n \times n$ deterministic matrix (called a variance profile) and $X_n = (X_{ij})$ a matrix with i.i.d. centred and unit variance entries. We consider the normalized matrix

$$Y_n = \left(\frac{1}{\sqrt{n}} \sigma_{ij} X_{ij} \right)$$

with eigenvalues $\lambda_i(Y_n)$. In this talk, we will describe the limiting behaviour of the associated spectral measure

$$\mu_n^Y = \frac{1}{n} \sum_{i=1}^{n} \delta_{\lambda_i(Y_N)}$$

as n goes to infinity.
An important feature of the presented result will be to consider variance profiles with possibly vanishing entries, as long as a quantitative irreducibility hypothesis is fullfilled.
The study of spectral properties of such matrices is of interest in the understanding of Lotka-Volterra systems, used to describe foodwebs dynamics.
Joint work with Nick Cook, Walid Hachem and David Renfrew. Preprint available at arXiv:1612.04428.

NEMISH Yuriy: Talk. "Location of the spectrum of Kronecker random matrices"
In this talk we will discuss certain spectral properties of a general class of large non-Hermitian random block matrices. We prove that for any such matrix there are no eigenvalues away from a deterministic set with very high probability. This set is obtained from the Dyson equation of the corresponding Hermitization matrix as the self-consistent approximation of the pseudospectrum. We demonstrate that the analysis of the matrix Dyson equation by Ajanki, Erdös and Krüger (2018) offers a unified treatment of many structured matrix ensembles. This is a joint work with Johannes Alt, László Erdös and Torben Krüger.

ROTA-NODARI Simona: Talk. "Renormalized Energy Equidistribution and Local Charge Balance in Coulomb Systems"
We consider a classical system of n charged particles confined by an external potential in any dimension bigger or equal than 2. The particles interact via pairwise repulsive Coulomb forces and the pair-interaction strength scales as the inverse of n (mean-field regime). The goal is to investigate the microscopic structure of the minimizers.
It has been proved by Sandier-Serfaty (d=2) and Rougerie-Serfaty (d>2) that the distribution of particles at the microscopic scale, i.e. after blow-up at the scale corresponding to

the interparticle distance, is governed by a renormalized energy which corresponds to the total Coulomb interaction of point charges in a uniform neutralizing background.
In this talk, I will present some results which show that for minimizers and in any large enough microscopic set, the renormalized energy concentration and the number of points are completely determined by the macroscopic density of points. In other words, points and energy are "equidistributed".
Works in collaboration with S. Serfaty and M. Petrache.

SPEICHER Roland: Mini-course. "On the Brown measure in Free Probability"
This mini-course will center around the notion of the Brown measure. This is a generalization of the eigenvalue distribution for a general (not necessarily normal) operator in a finite von Neumann algebra (i.e. a von Neumann algebra which possesses a trace). It was introduced by Larry Brown in 1983, but fell into obscurity soon after. It was revived by Haagerup and Larsen around 2000 and played an important role in Haagerup's investigations around the invariant subspace problem.
The relevance of the Brown measure in a random matrix context comes from the fact that for "nice" random matrix models the eigenvalue distribution of the random matrices converges to the Brown measure of the operator which is the limit of the random matrices in *-distribution.
Apart from the basic theory of the Brown measure we will in particular also address the calculation of the Brown measure for R-diagonal operators (which corresponds to the singular ring theorem in random matrix theory). We hope to convey the message that operator-valued free probability theory gives the right frame for the Brown measure.
A main reference for this course is Chapter 11 of `https://www.math.uni-sb.de/ag/speicher/publikationen/Mingo-Speicher.pdf`