Kinetic Theory & Approximation (24hrs)

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Francis Filbet will deliver a mini-course on analytical and numerical methods for kinetic equations and transport phenomena. In this course the design and the mathematical analysis of numerical methods for kinetic equations will be considered. Kinetic theory represents a way of describing the time evolution of a system consisting of a large number of particles. This class of models is an essential tool to make the rigorous link between a microscopic description and a macroscopic description of the physical reality. Furthermore, due to the high number of dimensions and their intrinsic physical properties, the construction of numerical methods represents a challenge and requires a careful balance between accuracy and computational complexity.

We will focus on a classical system dealing with Coulombian (or Newtonian) interactions between particles in the presence of a strong external magnetic field and present the analysis of the Vlasov-Poisson system (weak and strong solutions)

1 Transport equations

- Theory of characteristics for linear equation
- Existence and uniqueness of weak and classical solutions

2 The Vlasov-Poisson system [1]

- Existence of weak solutions : solutions à la Di Perna-Lions
- Averaging lemma : regularizing effects on transport equations - Application to the Vlasov-Maxwell system

3 Dispersion lemma

- Strichartz estimates on transport equations [3]

4 Approximation of transport equations

- Finite volume method [2]
- Stability and convergence results

References

[1] F. Bouchut, F. Golse, M. Pulvirenti, Kinetic equations and asymptotic theory, Series in Appl. Math., Gauthiers-Villars, 2000

[2] F. Bouchut, Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well- balanced schemes for sources, Frontiers in Mathematics series, Birkhauser, 2004

[3] Terence Tao, Nonlinear dispersive equations: local and global analysis, , CBMS regional conference series in mathematics