

# An introduction to vector bundles and $K$ -theory

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## Abstract:

The goal of this course is to provide an introduction to  $K$ -theory, according to the following agenda:

### Chapter 1. Vector Bundles.

1. Basic Definitions and Constructions. Sections. Direct Sums. Inner Products. Tensor Products. Associated Fiber Bundles.
2. Classifying Vector Bundles. Pullback Bundles. Clutching Functions. The Universal Bundle. Cell Structures on Grassmannians.

### Chapter 2. $K$ -Theory.

1. The groups  $K(X)$ . Ring Structure. The Fundamental Product Theorem.
2. Bott Periodicity. Exact Sequences. Deducing Periodicity from the Product Theorem. Extending to a Cohomology Theory. Elementary Applications.
3. Division Algebras and Parallelizable Spheres.  $H$ -Spaces. Adams Operations. The Splitting Principle.
4. Bott Periodicity in the Real Case. (Option)
5. Vector Fields on Spheres.

### Chapter 3. Characteristic Classes. (Only if time allows it)

1. Stiefel-Whitney and Chern Classes. Axioms and Constructions. Cohomology of Grassmannians.
2. Euler and Pontryagin Classes. The Euler Class. Pontryagin Classes.

## Prerequisites:

General topology, smooth manifolds and smooth maps.

## References:

1. HATCHER, ALLEN, *Vector bundles and  $K$ -theory*