

# Course A4 : Elliptic Partial Differential Equations and Evolution Problems

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## Abstract:

The aim of this lecture is to provide a background of methods and techniques for the analysis of elliptic partial differential equations and evolution problems and their numerical approximations.

1. Variational models: elasticity, least action principle, Euler-Lagrange equation.
2. Sobolev spaces in a domain of  $\mathbb{R}^n$ . Traces. Embeddings. Poincaré inequalities.
3. Elliptic partial differential equations in divergence form:
  - (a) Lax-Milgram lemma,  $H^2$ -regularity, spectral theory of the Laplace operator, maximum principle.
  - (b) Elliptic partial differential equations in divergence form: Galerkin approximations, finite elements methods.
4. Evolution problems:
  - (a) semigroups, Hille-Yosida theorem, linear parabolic and dispersive equations, Duhamel's formula.
  - (b) Dynamical systems: Stability, Liapunov functional, LaSalle Invariance principle.
  - (c) Nonlinear parabolic problems.

## Prerequisites:

$L^p$ -spaces. Hilbert spaces. Ordinary differential equations.

## References:

1. H. BREZIS. *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Universitext, Springer, 2011.
2. TH. CAZENAVE AND A. HARAUX. *Introduction aux problèmes d'évolution semi-linéaires*. Ellipses, 1990.
3. A. ERN AND J.L. GUERMOND. *Éléments finis : théorie, applications, mise en oeuvre*. Springer, 2002.
4. L.C. EVANS. *Partial Differential Equations*. Graduate Studies in Mathematics **19**, Amer. Math. Soc., 1998.