

Approximation of PDEs

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Overview. This course is organized as follows: 10 academic lessons, joint with 4 numerical labs with Matlab or FreeFEM++. Each slot (either lesson or lab) lasts 2 hours.

The introduction (2 slots) deals with first linear systems, second with the principles of the Finite Difference Method (FDM) for Ordinary Differential Equations (ODEs). The second part of the course (7 slots) consists in advanced material on the Finite Element Method (FEM) applied to elasticity, and on Mixed Finite Elements applied to incompressible fluid mechanics. In the last 5 slots, the last part of the course, numerical methods being used for the simulation of the propagation of electromagnetic waves are presented.

Part 1: Introduction

1. Linear systems, condition number, decomposition of matrices for solving linear systems (EVD, SVD), least squares.
2. the Finite Difference Method (FDM) for Ordinary Differential Equations (ODEs): stability, consistency, convergence.

Part 2: More on the Finite Element Method

- 3&4. The case of coupled system of PDEs: Example of the problem of linear elasticity, stemming from continuum mechanics: recalls and reminders on continuum mechanics, variational formulation, boundary conditions, existence and uniqueness of a solution. Example of finite element formulation for 2-D elasticity.
5. **Lab 1:** Finite element computations in 2-D elasticity (stress and strain)
- 5'. **Lab 1':** Introduction to FreeFem++
- 6&7&8. Mixed Finite Element Method: an introduction on the Stokes equation (the Stokes equations govern the flow of a steady, viscous, incompressible, isothermal, Newtonian fluid). Variational formulation, link with constrained optimization, inf-sup condition (discrete and continuous). Example of inf-sup stable and inf-sup unstable finite element spaces.
9. **Lab 2:** Numerical solution of the Stokes equation thanks to FreeFem++.

Part 3: Propagation models of electromagnetic waves

10. Maxwell equations in 2-D: hyperbolicity of the system, boundary conditions, energy conservation. The Yee scheme: time-domain discretization, stability by plane-wave analysis, numerical speed of propagation, link with mixed finite element.
11. Finite Volume Method (FVM) for Maxwell equations in 2-D. Centered fluxes scheme. L^2 -stability. Study of the Riemann problem. Upwind scheme.
- 12&13. : High order local approximation: Discontinuous Galerkin methods (GD). Variational formulation. Approximation of order p on triangles. Computation of the mass and stiffness matrices. Weak formulation of the boundary conditions. L^2 -stability of the GD method with a leap-frog scheme in the time domain.
14. **Lab 3:** Numerical modelling of a waveguide.

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