Stability analysis of ODE's and PDE's periodic solutions.
Theoretical and Numerical Aspects.

Periodic patterns are ubiquitous in nature. They may appear as solutions of either ODE systems (Van Der Pol Oscillators) or PDE systems. In this later case, periodicity can be either spatial or in time and we shall focus on particular patterns, namely periodic travelling waves.

In a first part, we will consider periodic solutions of ODE's system through various examples and tackle the problem of existence (numerically and theoretically) and stability of periodic waves both from the spectral/linear and nonlinear point of view. At the linear level, we will introduce the Floquet theory for linear ODE system with time periodic coefficients. At the nonlinear level, we will introduce the notion of orbital stability and study some examples of planar ODE systems.

In a second part, we will consider the stability of periodic travelling waves. Here the theory is more involved and we will focus on spectral stability of periodic travelling waves: we will introduce Bloch decomposition and show how to tackle the problem from a numerical point of view with Hill's method. We also introduce the notion of modulational stability and consider some formal techniques (Whitham's modulation equations) to study this problem.

References

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